CALCULUS FOR BUSINESS AND SOCIAL SCIENCES COREQUISITE WORKBOOK

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Preface

Applied Calculus Corequisite Workbook

Victoria Dominguez, Cristian Martinez, and Sanaa Saykali

This document serves as a Corequisite Workbook for an Applied Calculus course. It is vitally important for students who take this course (and its corequisite) to have a thorough and complete refresher of Algebra skills needed for Calculus. These problems may be assigned as in-class practice problems, and can be also assigned as homework, to be graded and counted toward the Applied Calculus corequisite final grade.

Supported in part by the Academic Senate for California Community College's Open Educational Resources Initiative.

INTRODUCTION TO THE CO-REQUISITE CLASSROOM

So, what is meant by a student-centered learning environment and how does it differ from the traditional math classroom? In this learning model, the focus of activity shifts from the teacher to the learner. Class time is spent on discussion, collaborative work, and engagement with other brains-on activities. Additionally, during class, teaching and learning is tailored to fit the needs of small groups of students as they work through these activities and review prerequisite skills in a just-in-time approach. This learning model employs a teacher-guided-discovery process that allows the instructor to identify gaps in student understanding and use class time to remediate those gaps before moving on to the next topic. In summary, the student should expect to engage in the following activities during class.

- Group-work to motivate and introduce key concepts for each of the course topics
- Group-work to review skills and work through skill-building problems from topics previously discussed in class
- Class discussions
- Mini lectures as necessary to close gaps in concept mastery or skill attainment
- Mini on-the-spot Round Robin poster presentations
- Peer-review feedback
- Collaborate with classmates to master transfer-level Applied Calculus

Furthermore, the student should expect to do the following outside of class:

- Complete the assigned reading and homework
- Review course material daily
- Work through review problems to prepare for exams and quizzes
- Write reading responses

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Licensing

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CHAPTER OVERVIEW

1: The Real Numbers and the Number Line

1.1: Definition of Real Numbers and the Number Line

1.2: Number comparisons using < ,>, and =

Thumbnail: The addition 1+2 on the real number line. (CC BY-SA 3.0; Stephan Kulla);

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1.1: Definition of Real Numbers and the Number Line

Definition: Real Numbers

Real numbers are the numbers that are normally used in real world math problems.

Here are common groups of numbers that are real numbers:

Whole Numbers:	$0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ ...$	Positive counting numbers plus zero
Integers:	$...\ -3,\ -2,\ -1,\ 0,\ 1,\ 2,\ 3,\ ...$	Positive and negative whole numbers
Rational Numbers:	$13,\ \frac{2}{7},\ \frac{-1}{3},\ -2,\ 1.32,\ -12.64$	Numbers that may be written as a b , where a and b are integers. Decimals are rational numbers.
Irrational Numbers:	$e,\ \sqrt{8},\ -\sqrt{11},\ \pi,\ 0.1234$	Numbers that can't be expressed as a b . Irrational numbers are numbers with non-repeating and never-ending decimals!

Note: Real numbers may be positive or negative and include 0 as shown above.

Definition: The Number Line

A line that extends horizontally with coordinates that correspond to real numbers. The number line helps measure the distance between the origin (0) to a real number. Here's an example of a number line:

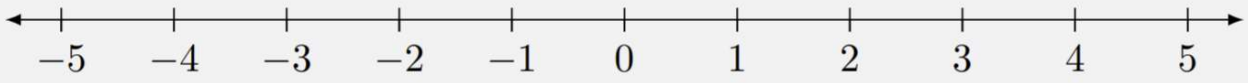

Figure 1.1.1

1.1.1: Reading the number line:

The origin corresponds to the number 0 in the number line.

To the left of the origin are the negative numbers.

To the right of the origin are the positive numbers.

Exercise 1.1.1

Graph the following numbers on the number line below: $-5,\ e,\ 3.5,\ -2.25,\ 7.01,\ -5.2,\ \sqrt{20},\ \pi$.

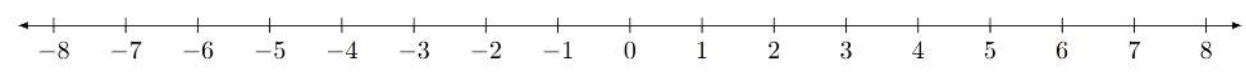

Figure 1.1.2

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1.2: Number comparisons using < ,>, and =

The number line is to compare numbers. Here are the symbols used when comparing two or more numbers:

Symbols used	Read as
$<$	less than
$\leq$	less than or equal to
$>$	greater than
$\geq$	greater then or equal to
$=$	equals

Caution: $\leq$ and $\geq$ are used if the numbers that are being compared satisfy at least one of the two conditions.

Note: Moving further to the left of the origin, the numbers are decreasing in value. Moving further to the right of the origin, the numbers are increasing in value.

Examples 1.2.1

Statement	Reason
$-3 < -2$	Since -3 is further to the left of -2
$5 > 1$	Since 5 is further to the right of 1
$2 > -6$	Since 2 is further to the right of -6
$3 \geq 3$	3 is not greater than 3, but they are equal!

Hint: The symbol $>$ is pointing to the right direction. The numbers in the right direction are increasing so use this symbol when the first number is greater than the second number. Think similarly for less than.($<$ pointing left so less than)

Exercise 1.2.1

For the following exercises compare the following numbers and fill in the blank by using the appropriate symbol $<$, $>$, $\geq$, $\leq$ or $=$

1. $-1____4$
2. $-4____9$
3. $0____-2$
4. $3____3$
5. $-12____12$
6. $4____6$
7. $-1____-1$
8. $5____-2$
9. $35____53$
10. $-29____-37$

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CHAPTER OVERVIEW

2: Cartesian Coordinate System

Mathematical figures in algebra are drawn in a system or plane called the **rectangular coordinate system**. Figures are particularly important for giving a visual representation of relationships between two variables.

2.1: Definition and Label Quadrants
2.2: Ordered Pairs
2.3: The Distance Formula
2.4: Applied Examples

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2.1: Definition and Label Quadrants

Definition: Rectangular Coordinate System

The **Rectangular Coordinate System** is also known as the **Cartesian Coordinate System**, is based on a grid and formed by two perpendicular number lines, the horizontal number line called the **x-axis** and the vertical number line called the **y-axis**. Every point on the rectangular coordinate system can be identified by unique x and y coordinates.

The two perpendicular number lines intersect at the point $(0, 0)$ and is called the origin. The rectangular coordinate system is divided into four regions. The regions are called quadrants and labeled in roman numerals as follows: **Quadrant I**, **Quadrant II**, **Quadrant III**, and **Quadrant IV**, as shown in the figure below

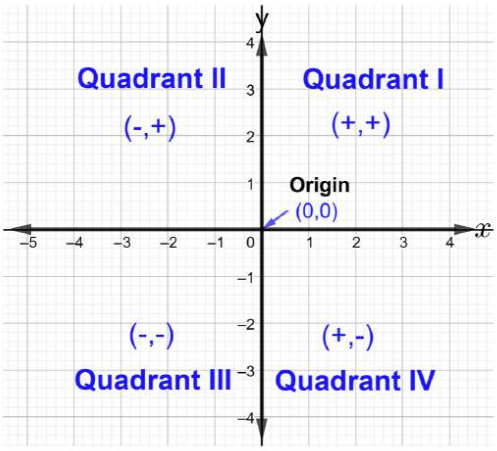

Figure 2.1.1

Every point in the coordinate plane corresponds to a unique ordered pair (x, y), where x and y are any real numbers. The ordered pair always have the x-coordinate first (on the left) and the y-coordinate second (on the right). $(6, 4)$, $\left(-.33, \frac{1}{5}\right)$, $(100, -2)$, $\left(\sqrt{3}, \frac{1}{2}\right)$ are few examples of ordered pairs. Read the ordered pair $(6, 4)$ as "the point six, four" where 6 is the x-coordinate and 4 is the y-coordinate.

Exercise 2.1.1

1. What are points called that are of the form (x, y)?
2. Identify the coordinates of the following ordered pairs: $(-10, 0)$, $(0.5, -6)$, $\left(\frac{1}{2}, -\frac{5}{3}\right)$
3. Name and state the ordered pair of the point where the perpendicular axes of the rectangular coordinate system intersect.
4. State the 4 regions in the rectangular coordinate system.
5. Name the first coordinate in an ordered pair.
6. Name the second coordinate in an ordered pair.
7. What is the **horizontal** number line in the rectangular coordinate system?
8. What is the **vertical** number line in the rectangular coordinate system?

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2.2: Ordered Pairs

Definition: Ordered Pairs

Ordered pairs are pairs of numbers used to locate a point in the rectangular coordinate plane and written in the form (x, y), where x is the x-coordinate and y is the y-coordinate.

The location of the ordered pair in the quadrants will determine the sign of the x and y coordinates, as shown in the previous section, figure above. Thus, the signs of the ordered pairs (x, y) are summarized as follows,

- If (x, y) is in *Quadrant I* then both x and y are positive.
- If (x, y) is in *Quadrant II* then x is negative and y is positive.
- If (x, y) is in *Quadrant III* then both x and y are negative.
- If (x, y) is in *Quadrant IV* then x is positive and y is negative.

Note: Points on the perpendicular axis in a rectangular plane do not belong to any of the quadrants.

To locate a point in the coordinate plane, use a process called **graphing** a point. This process can also be called **plotting** a point. Always start at the origin and move right or left to locate the x-coordinate and move up or down to locate the y-coordinate.

To graph or plot $(-2, 5)$ on a coordinate plane, start at the origin and move 2 units to the left, because the x-coordinate -2 is negative, then move up 5 units because the y-coordinate 5 is positive and draw a dot. notice that our point is in quadrant II. To graph the point $(4, 1)$, start at the origin and move 4 units to the right and 1 unit up. Similarly, graph the points $(-1, -1)$ and $(2, -3)$, as shown in the figure below.

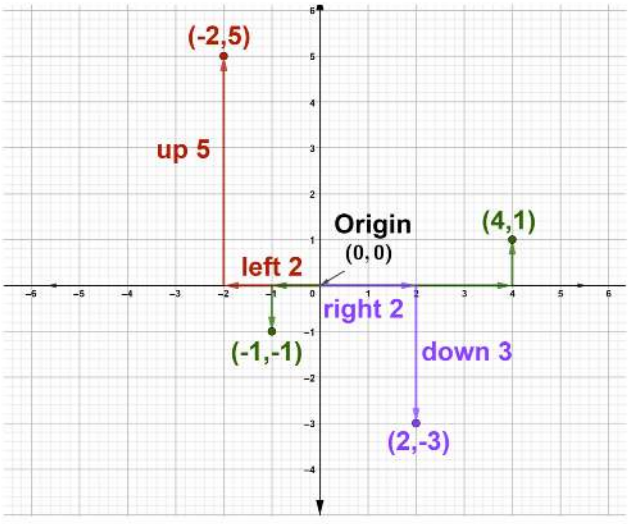

Figure 2.2.1

Careful! In the previous chapter the notation (p, q) was used to represent an absolute value inequality solution in interval notation. This section introduces ordered pairs that use the same parentheses representation. Make sure not to confuse the interval notation with the ordered pairs notation.

Example 2.2.1

Graph the given points on the same rectangular coordinate plane, and state where each point lies. If a point does not lie in any of the quadrants, state the axis on which it lies.

a. $(3, 4)$
b. $(-2, -2)$
c. $\left(0, \frac{5}{2}\right)$
d. $(-3.5, 0)$

Solution

a. Both x and y coordinates are positive. start at the origin, move 3 units to the right then, move up 4 units and mark the point $(3, 4)$ The point lies in quadrant I

b. Both x and y coordinates are negative. start at the origin, move 2 units to the left, then, move 2 units down and mark the point $(-2, -2)$. The point lies in quadrant II.

c. Since the x-coordinate is 0, start at the origin and don't move right or left. Since the y-coordinate is positive, move $\frac{5}{2}$ units or 2.5 units up. Label point $\left(0, \frac{5}{2}\right)$. The point lies on the y-axis.

d. Sine the x-coordinate is negative, start at the origin and move 3.5 units to the left. Since the y-coordinate is 0, don't move up or down. label the point $(-3.5, 0)$. The point lies on the x-axis.

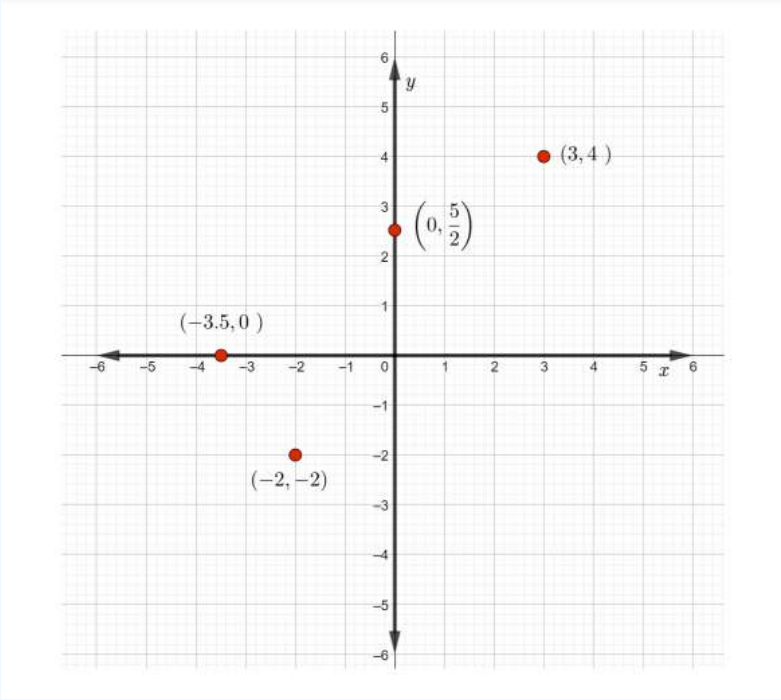

Figure 2.2.2

Example 2.2.2

Determine the coordinates of the points labeled in the figure below.

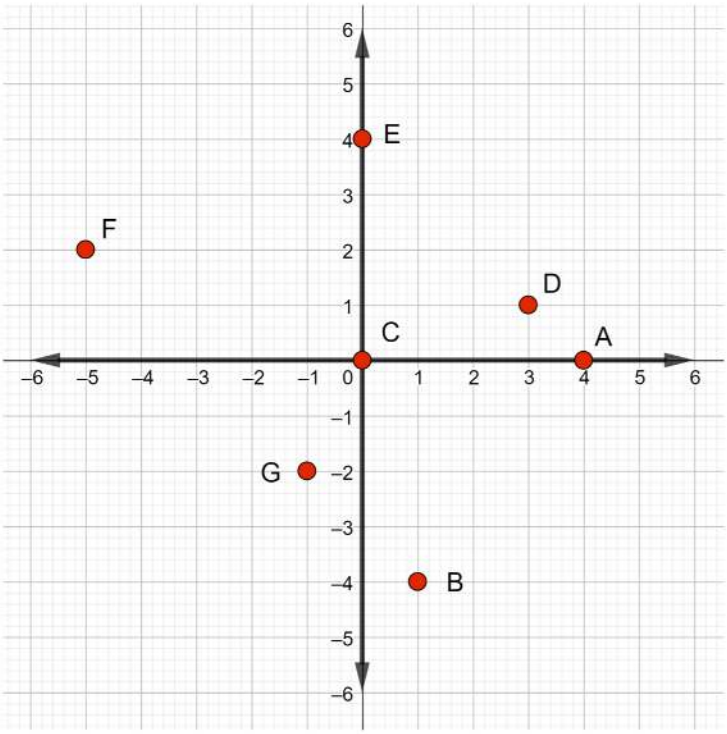

Figure 2.2.3

Solution

To plot any point in the coordinate plane, start at the origin then move according to the sign of the given coordinates as follows,

- **Point A:** Start at the origin and move 4 units to the right. The x-coordinate is 4. There is no vertical movement up or down, so the y-coordinate is 0. Hence, Point A has coordinates $(4, 0)$
- **Point B:** Again, start at the origin and move 1 unit to the right and 4 unit down. Hence, point B have coordinates $(1, -4)$
- **Point C:** Since the x-coordinate is 0, then start at the origin and don't move right or left. The y-coordinate is 0 as well, so don't move up nor down. Thus, the coordinates of point C are $(0, 0)$
- **Point D:** From the origin, move 3 units to the right, then 1 unit up. Thus, coordinates of point D are $(3, 1)$
- **Point E:** Start at the origin and stay there because the x-coordinate is 0, then move 4 units up. Thus, Point E have coordinates $(0, 4)$.
- **Point F:** Move 5 units left of the origin and 2 units up. Point F have coordinates $(-5, 2)$
- **Point G:** Move 1 unit left of the origin and 2 units down. Point G have coordinates $(-1, -2)$

Exercise 2.2.1

1. What are the x and y coordinates in $(-3, 2)$ and $(2, -3)$? Are they the same Point?
2. Plot the following points on the same rectangular coordinate plane, and state the quadrant in which each point lies or the axis on which it lies. $(-1.5, 2)$, $(0, -3)$, $(5, 2.5)$, $\left(-\frac{1}{2}, -\frac{1}{2}\right)$, $\left(3\frac{1}{2}, -\frac{7}{2}\right)$, $(-3, 3)$, $(-2, 0)$
3. Determine the coordinates for each given point in the figure below.
4. Determine the coordinates for each given point in the figure below.

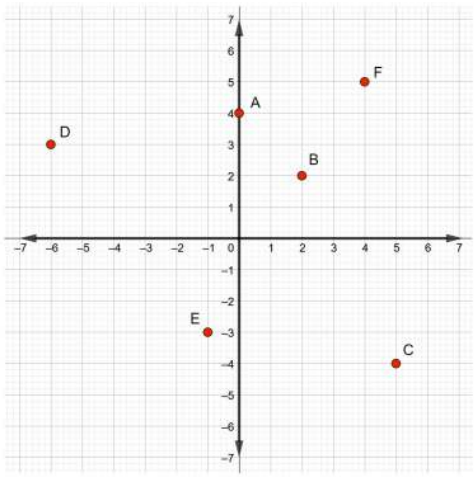

Figure 2.2.5

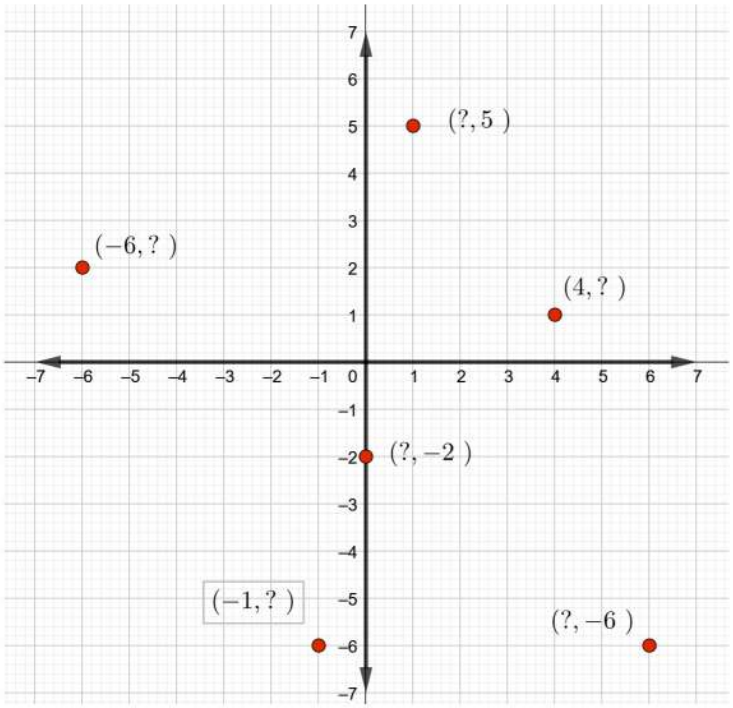

Figure 2.2.4

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2.3: The Distance Formula

The previous section taught how to plot points in the rectangular coordinate plane. This section teaches how to find the distance between any two points in the plane. For example, to find the distance of points (x_1, y_1) and (x_2, y_2) consider the following formula:

Definition: Distance Formula

The distance d between two points, $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ in the plane is given by:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)}^2$$

Example 2.3.1

Find the distance between the points $(-5, 2)$ and $(3, 4)$

Solution

Let $P_1(-5, 2)$ and $P_2(3, 4)$ be two points in the plane and let $x_1 = -5$, $y_1 = 2$, $x_2 = 3$, and $y_2 = 4$.

Using the distance formula with the given values:

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3 - (-5))^2 + (4 - 2)^2} \\ &= \sqrt{(3 + 5)^2 + (2)^2} \\ &= \sqrt{8^2 + 2^2} \\ &= \sqrt{64 + 4} \\ &= \sqrt{68} \\ &= 2\sqrt{17} \end{aligned}$$

Therefore, the distance between the two given points is $2\sqrt{17}$.

Example 2.3.2

Find the distance between the points $(-2.5, -1)$ and $(-3, -1.5)$.

Solution

Let $P_1(-2.5, -1)$ and $P_2(-3, -1.5)$ be points in the plane and let $x_1 = -2.5$, $y_1 = -1$, $x_2 = -3$ and $y_2 = -1.5$.

Then using the distance formula with the given values yields,

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[-3 - (-2.5)]^2 + [-1.5 - (-1)]^2} \\ &= \sqrt{(-3 + 2.5)^2 + (-1.5 + 1)^2} \\ &= \sqrt{(-0.5)^2 + (-0.5)^2} \\ &= \sqrt{0.25 + 0.25} \\ &= \sqrt{0.5} \\ &\approx 0.71 \end{aligned}$$

Therefore, the distance between the two given points is approximately 0.71.

Exercise 2.3.1

1. Find the distance between $P_1(-3, -1.5)$ and $P_2(-2.5, -1)$. Compare the answer to the answer in example 2. What can be concluded?
2. Find the distance between $(-3, 6)$ and $(2, 4)$

3. Find the distance between the points $\left(\frac{1}{2}, -\frac{10}{4}\right)$ and $\left(-\frac{14}{4}, -\frac{5}{2}\right)$

4. Why is the distance formula used?

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2.4: Applied Examples

In this section, apply the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to find the lengths of line segments.

Note: Three points A, B, and C are collinear, or in other words, the three points lie on the same line, if the sum of the lengths of any two line segments connecting the points, is equal to the length of the remaining line segment. That is, $AB + BC = AC$ or, $AB + BC = AC$ or, $AB + AC = BC$ or $AC + BC = AB$.

Example 2.4.1

Determine if the given three points are collinear.

$$A(10, -4) \quad B(8, -2) \quad C(2, 4)$$

Solution

First find segments AB, BC, and AC. To do so, find the distance between the points A and B, B and C, A and C.

$$\begin{aligned}
\text{Segment AB} &= \text{The distance between point A and Point B} \\
&= \sqrt{(8-10)^2 + [-2-(-4)]^2} \\
&= \sqrt{(-2)^2 + (2)^2} \\
&= \sqrt{8} \\
&= 2\sqrt{2} \\
\text{Segment BC} &= \text{The distance between point B and Point C} \\
&= \sqrt{(2-8)^2 + [4-(-2)]^2} \\
&= \sqrt{(-6)^2 + (6)^2} \\
&= \sqrt{72} \\
&= 6\sqrt{2} \\
\text{Segment AC} &= \text{The distance between point A and Point C} \\
&= \sqrt{(2-10)^2 + [4-(-4)]^2} \\
&= \sqrt{(-8)^2 + (8)^2} \\
&= \sqrt{128} \\
&= 8\sqrt{2}
\end{aligned}$$

Thus,

$$\begin{aligned}
AB + BC &= 2\sqrt{2} + 6\sqrt{2} \\
&= 8\sqrt{2} \\
&= AC
\end{aligned}$$

Since Thus, $AB + BC = AC$ then three points are collinear.

Exercise 2.4.1

1. Determine if the following points are collinear.
   a. $A(4, -1) \quad B(5, -2) \quad C(1, 2)$
   b. $A(2, -2) \quad B(3, 1) \quad C(2, 1)$

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CHAPTER OVERVIEW

3: Interval Notation

Interval notation is used to describe a group of numbers in the number line. In mathematics these groups are called Sets. The interval notation is used to group all the numbers between two numbers being studied. The notation used to describe a set are the following.

Symbols used	
(	used to start interval
[	used to start interval
)	used to end interval
]	used to end interval

These symbols can be used in different combinations. The combinations have meaning and this section will teach how to successfully use these symbols to describe any desired set. **Note:** number on the left side should always be less than the number on the right side (Incorrect : (5,2)).

3.1: [a, b]
3.2: [a, b)
3.3: (a, b]
3.4: (a, b)
3.5: Interval Notation and Infinity

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3.1: [a, b]

This section will focus on intervals in the form [,]. By observation, the interval begins and ends with brackets. The use of brackets implies that the endpoints are being included in the set. This section will include the endpoint on the right and the endpoint on the left since again the interval notation starts and ends with brackets.

Example 3.1.1

$[2, 5]$

Solution

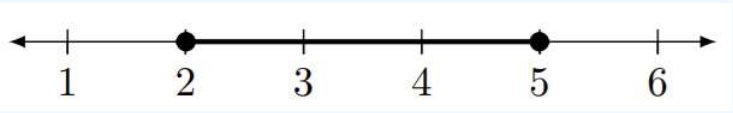

Figure 3.1.1

In this example , the set includes the endpoints 2 and 5, the set also includes every number between 2 and 5. Hence, 3, 3.25, 4, 4.9999, and 5 are all part of the set. When graphing the set in the number line closed bullets • are used in the endpoints that are included in the set.

Example 3.1.1

$[-4, 10]$

Solution

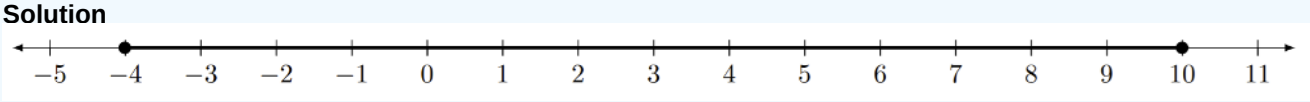

Figure 3.1.2

Similar to the previous example, the endpoints -4 and 10 are included in the set. The set contains all numbers between -4 and 10. Hence, the numbers -2, 0, 1, 2, 3, 4.5, 10 are some examples of numbers inside the interval $[-4, 10]$.

Exercise 3.1.1

Draw a number line for the following intervals and list at least three numbers inside the set.

1. $[-3, 3]$
2. $[-23, -20]$
3. $[-101, 242]$
4. $[2.5, 3]$
5. $[0, 12]$
6. $[-4.2, 6.1]$

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3.2: [a, b)

This section will study intervals in the form $[a, b)$. The interval begins with a left bracket and ends with a right parentheses. Starting the interval with a brackets means the right endpoint is included in the set and ending with the use of a parentheses means the left endpoint in not included.

Example 3.2.1

$[2, 5)$

Solution

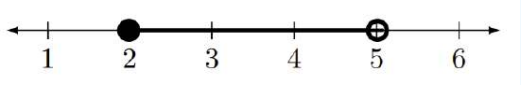

Figure 3.2.1

In the example above, the interval starts with a left bracket and ends with a parentheses. Hence,the interval includes the number 2 and does not include the number 5. By observing the corresponding number a closed bullet • is used to demonstrate that the number 2 is included in the interval, on the other hand an open bullet ◦ is on the number 5 to demonstrate that 5 in not in the interval. Some of the numbers in the set include 2, 2.5, 3.4, 4 and 4.99. Again, the number 5 is not in this set.

Example 3.2.2

$[-4, 10)$

Solution

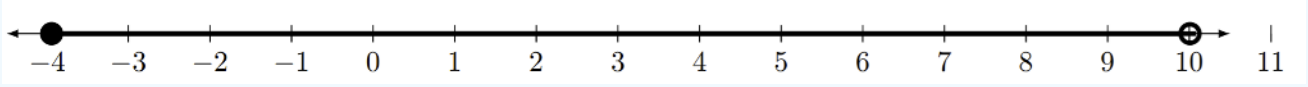

Figure 3.2.2

Similar to example 1, the endpoint on the left which corresponds to the number -4 is included in the interval, while the endpoint on the right is not included in the interval. Some numbers on the interval above are -4, -2, 0, 3.5, 8.5 and 9.9.

Exercise 3.2.1

Draw a number line for the following intervals and list at least three numbers inside the set.

1. $[-3, 1)$
2. $[-12, -2)$
3. $[-233, -223)$
4. $[3.5, 5)$
5. $[0, 1)$
6. $[-1.4, 2.8)$

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3.3: (a, b]

Be careful with how the interval notation is written! The notation in this section looks similar to the previous section, but they are not same. By looking at the notation closer it becomes clear that the endpoint on the left is not included in the set while the endpoint on the right is included in the interval.

Example 3.3.1

$(2, 5]$

Solution

Figure 3.3.1

The left endpoint, which corresponds to number 2 is not on the interval. The right endpoint, which corresponds to number 5 is included. Examples of numbers in this set is 3, 4, 4.5 and 5

Example 3.3.2

$(-4, 10]$

Solution

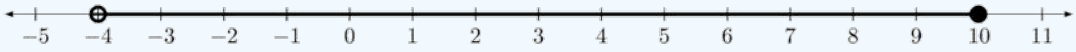

Figure 3.3.2

The endpoints of this interval behave the same way as example 1. When graphing, remember to place • when endpoint is included and ◦ when the endpoint is not included. Some examples of numbers in this interval are -3.99, -2, 0 and 10.

Exercise 3.3.1

Draw a number line for the following intervals and list at least three numbers inside the set.

1. $(6, 12]$
2. $(-31, -20]$
3. $(-200, -199]$
4. $\left(\frac{3}{4}, 5\right]$
5. $\left(\frac{-11}{2}, \frac{7}{2}\right]$
6. $(0.15, 6.95]$

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3.4: (a, b)

In this section, both endpoints are not included in the interval.

Example 3.4.1

$(2, 5)$

Solution

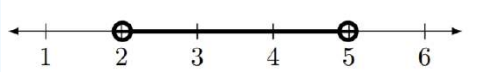

Figure 3.4.1

The interval above includes all numbers between 2 and 5, but because the interval has parentheses on both the left and the right sides of the interval, the end points are not included.

Example 3.4.2

$(-4, 10)$

Solution

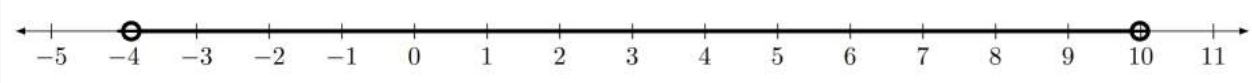

Figure 3.4.2

Similar to the explanation for the previous examples, the end points are not included in the interval. So the interval includes every number between -4 and 10, but not -4 and 10.

By now, students should have a strong idea of the numbers that are included and not included in the examples above. Students should also be able to analyze any given number line and write the appropriate interval notation for that number line. Try the following exercises to check for understanding.

Exercise 3.4.1

Draw a number line for the following intervals and list at least three numbers inside the set.

1. $(0, 1)$
2. $(-11, -5)$
3. $(100, 325)$
4. $(5, 6)$
5. $(12.1, 26.55]$

Exercise 3.4.2

Draw a number line that corresponds to the following intervals.

1. $[-2, 3)$
2. $[-5, -2]$
3. $(0, 5]$
4. $(-3, 4)$
5. $[-3, -2]$
6. $(-7, 2.5)$

Exercise 3.4.3

Write the appropriate interval notation for the given number lines.

1.

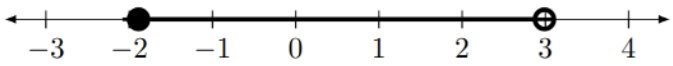

Figure 3.4.3

2.

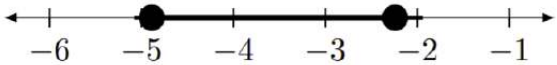

Figure 3.4.4

3.

Figure 3.4.5

4.

Figure 3.4.6

5.

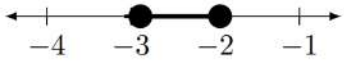

Figure 3.4.7

6.

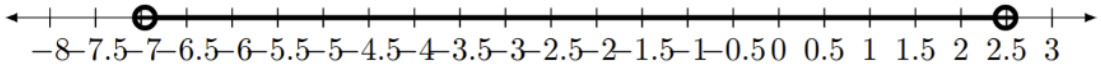

Figure 3.4.8

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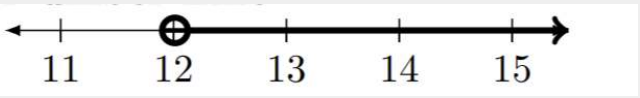
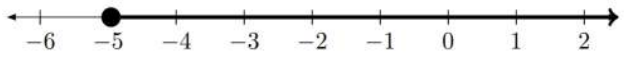
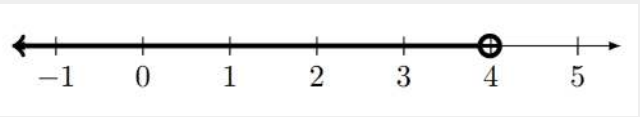
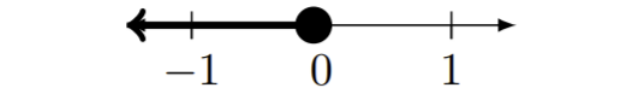
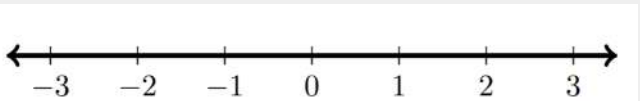
3.5: Interval Notation and Infinity

What is infinity? Infinity is not a real number. Infinity is larger than any number that can be imagined. It is an idea of having no bounds. A line is an example of having no bounds. For instance the number line has arrows at the end to represent this idea of having no bounds. The symbol used to represent infinity is ∞. On the left side of the number line is $-\infty$ and on the right side of the number line is ∞ to describe the boundless behavior of the number line.

Interval Notation	Number Line
a) $(12, \infty)$	 Figure 3.5.1
b) $[-5, \infty)$	 Figure 3.5.2
c) $(-\infty, 4)$	 Figure 3.5.3
d) $(-\infty, 0]$	 Figure 3.5.4
e) $(-\infty, \infty)$	 Figure 3.5.5

Note: Since ∞ is not a real number, it is required to use parentheses (,). ∞ can't be included in the interval.

? Exercise 3.5.1

Draw a number line that corresponds to the following intervals.

a. $(-\infty, 5)$
b. $[-5, \infty)$
c. $(-\infty, \infty)$
d. $(-3, \infty)$
e. $(-\infty, -2]$

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CHAPTER OVERVIEW

4: Functions

4.1: Function Definition
4.2: Function Notation
4.3: Evaluating a Function
4.4: Linear Functions
4.5: Absolute-Value Functions
4.6: Polynomial Functions
4.7: Domain and Range of a Function
4.8: Graphing Functions (without using Calculus)
4.9: Function Composition
4.10: Finding all Real Roots of a Function
4.11: Piecewise-Definition Functions
4.12: Applied Examples of Functions

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4.1: Function Definition

Definition: Function

A function is a rule that assigns to each element in the set of input values (the domain), one and only one element in the set of output values (the range).

Example 4.1.1

Determine if each of the following equations are functions:

a. $y = x^2 + 1$
b. $y^2 = x + 1$

Solution

a. To see the result of this equation, let x = 3.

$$\begin{aligned} y &= x^2 + 1 \\ y &= 3^2 + 1 \\ y &= 9 + 1 \\ y &= 10 \end{aligned}$$

Any value entered for x yields exactly one value for y.

There is only one solution for y, $y = 10$.

$y = x^2 + 1$ is a function!

b. To see the result of this equation, once again let $x = 3$.

$$\begin{aligned} y^2 &= x + 1 \\ y^2 &= 3 + 1 = 4 \\ y &= \sqrt{4} \\ y &= 2 \text{ or } y = -2 \end{aligned}$$

Any value entered for x will not yield exactly one value for y. There are two solutions for y, $y = 2$ and $y = -2$.

$y^2 = x + 1$ is NOT a function!

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4.2: Function Notation

Functions are written as "$f(x)$= an algebraic expression". Since $y = f(x)$, $f(x)$ is the same thing as y. This notation expresses x as the input into the function, and $f(x)$ as the output from the function.

$f(x)$ does **NOT** mean $f * x$!!

The most common function names are $f(x)$, $g(x)$ or $h(x)$, although others may be used such as $j(x)$, $k(x)$ or $m(x)$.

Most of the time, a function is called $f(x)$.

There are no examples or homework in this section.

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4.3: Evaluating a Function

When a function is evaluated, replace x with a given numeric value or an algebraic expression, and then simplify the result.

Example 4.3.1

Given $f(x) = -x^2 + 5x + 12$, find each of the following:

a. $f(2)$
b. $f(-5)$
c. $f(t)$
d. $f(4x - 1)$

Solution

a. Replace x with 2:

$f(x) = -x^2 + 5x + 12$	Given equation
$f(2) = -(2^2) + 5 * 2 + 12$	Replace x with 2 - notice that only 2 is squared, not the minus sign
$f(2) = -4 + 10 + 12$	Simplify
$f(2) = 18$	Solution

b. Replace x with -5:

$f(x) = -x^2 + 5x + 12$	Given equation
$f(-5) = -((-5)^2) + 5 * (-5) + 12$	Replace x with -5 - notice that it's $-x^2$, so the -5 is squared, but the result is still ne
$f(2) = -25 + (-25) + 12$	Simplify
$f(2) = -38$	Solution

c. Replace x with t:

$f(x) = -x^2 + 5x + 12$	Given equation
$f(t) = -t^2 + 5 * (t) + 12$	Replace x with t
$f(t) = -t2 + 5t + 12$	Simplify

d. Replace x with $(4x - 1)$:

$f(x) = -x^2 + 5x + 12$	Given equation
$f(4x - 1) = -(4x - 1)^2 + 5 * (4x - 1) + 12$	Replace x with $(4x - 1)$
$f(4x - 1) = -(16x^2 - 8x + 1) + 20x - 5 + 12$	Expand$(4x - 1)^2$ and distribute the 5 to $(4x - 1)$
$f(4x - 1) = -16x^2 + 8x - 1 + 20x - 5 + 12$	Distribute the negative sign
$f(4x - 1) = 16x^2 + 28x + 6$	Simplify

Exercise 4.3.1

1. For the function $f(x) = x^3 - 9$, find
   a. $f(3)$
   b. $f(2x - 5)$
   c. $f(t)$
2. For the function $f(x) = x^2 - 4x + 1$, find
   a. $f(2)$
   b. $f(4x + 3)$
   c. $f(z)$
3. For the function $f(x) = x^4 + 9x^2 - 6x - 1$, find
   a. $f(1)$
   b. $f(x + 2)$
   c. $f(a)$

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4.4: Linear Functions

Definition: Linear Function

A Linear Function is a function that has the form $f(x) = mx + b$. Any line that can be expressed in the form $y = mx + b$ is also a function.

Use function notation when an equation of a line written in Slope-Intercept form has no gaps or breaks and the line is not a vertical line. Linear functions written as $f(x) = mx + b$ pass the vertical line test:

Definition: Vertical Line Test

The Vertical Line Test is used to determine whether a graph defines the vertical output as a function of the horizontal input. If any vertical line would cross the graph more than once, then the graph does not define only one vertical output for each horizontal input.

For more information about linear equations, refer to the section on Straight Lines.

Example 4.4.1

Create a table of solutions and graph the following linear functions:

$f(x) = 2x - 3$

Solution

$f(x) = 2x - 3$

To find two ordered pairs, choose small values of x, then compute values of $f(x)$.

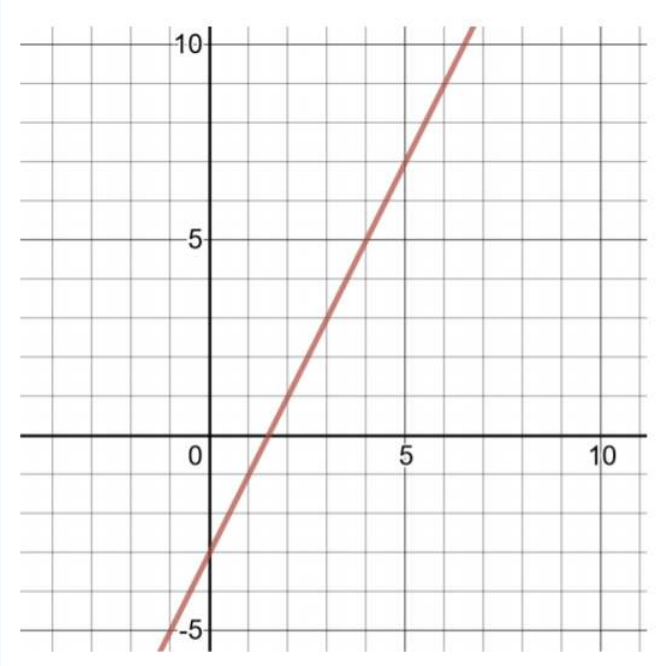

Figure 4.4.1

Table of Solutions for $f(x) = 2x - 3$	
x	$f(x)$
-1	$f(-1) = 2(-1) - 3 = -2 - 3 = -5$

0	$f(0) = 2(0) - 3 = 0 - 3 = 3$

Example 4.4.2

Create a table of solutions and graph the following linear function:

$g(x) = \frac{1}{3}x + 4$

Solution

To find two ordered pairs, choose small values of x, then compute values of $g(x)$. Because the coefficient of the term containing x is a fraction, choose multiples of the denominator for the product of $\frac{1}{3}x$ to be an integer.

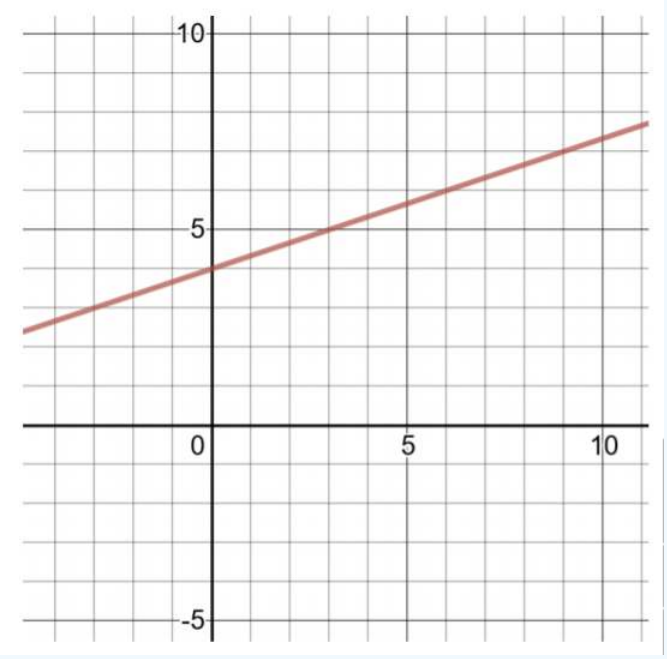

Figure 4.4.2

Table of Solutions for $g(x) = \frac{1}{3}x + 4$	
x	$g(x)$
0	$g(0) = \frac{1}{3}(0) + 4 = 4$
3	$g(3) = \frac{1}{3}(3) + 4 = 1 + 4 = 5$

Example 4.4.3

Create a table of solutions and graph the following linear functions:

$h(x) = -4x - 1$

Solution

To find two ordered pairs, choose small values of x, then compute values of $h(x)$.

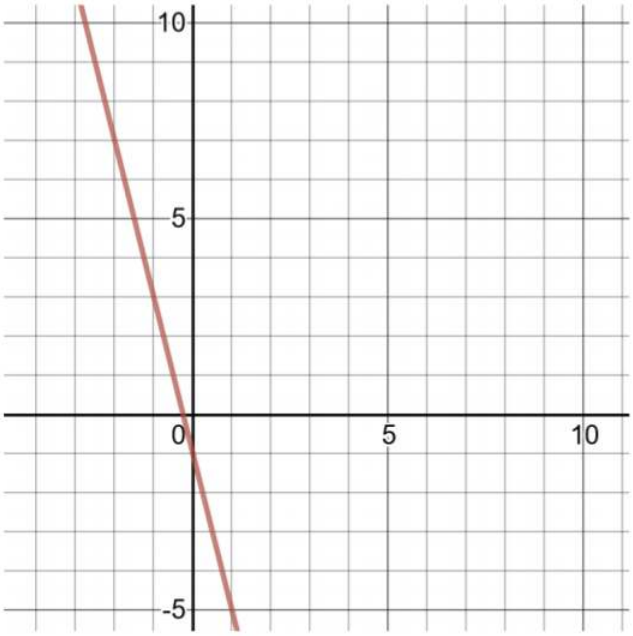

Figure 4.4.3

Table of Solutions for $h(x) = -4x - 1$	
x	$h(x)$
0	$h(0) = -4(0) - 1 = -1$
1	$h(1) = -4(1) - 1 = -5$

Example 4.4.4

Create a table of solutions and graph the following linear functions:

$h(x) = -\frac{3}{4}x - \frac{1}{4}$

Solution

To find two ordered pairs, choose small values of x, then compute values of $h(x)$.Because the coefficient of the term containing x is a fraction, choose multiples of the denominator for the product of $-\frac{3}{4}x$ to be an integer.

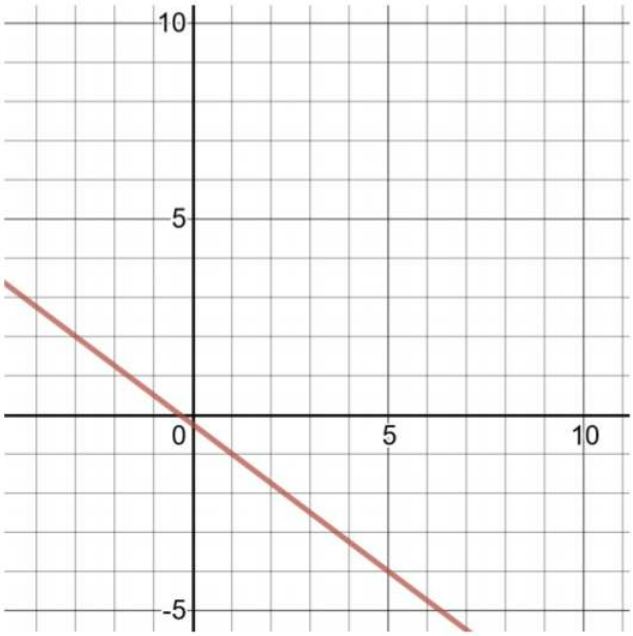

Figure 4.4.4

Table of Solutions for $h(x) = -\frac{3}{4}x - \frac{1}{4}$	
x	$h(x)$
0	$h(0) = -\frac{3}{4}(0) - \frac{1}{4} = -\frac{1}{4}$
4	$h(4) = -\frac{3}{4}(4) - \frac{1}{4} = -3 - \frac{1}{4} = -3\frac{1}{4}$

Exercise 4.4.1

Create a table of solutions and graph the following linear functions:

1. $f(x) = 4x - 9$
2. $g(x) = \frac{1}{2}x - 2$
3. $h(x) = -3x + 5$
4. $f(x) = -\frac{2}{3}x - \frac{1}{3}$

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4.5: Absolute-Value Functions

To graph absolute-value functions, choose small values of x, and compute the value of $f(x)$ from the given function to create ordered pairs. Three ordered pairs is the minimum amount needed to graph an absolute value function. Be careful, because one ordered pair must represent the vertex, the point where the left and right sides of the function meet. To correctly graph the shape of the absolute-value function, the vertex must be found.

$f(x) = a|x - h| + k$ General Form of an Absolute-Value Function, with vertex (h, k)

- a determines both the width and the orientation (facing up or down) of the function.
- h is the horizontal shift from the origin.
- k is the vertical shift from the origin.

Start by identifying the ordered pair of the vertex, and then find one ordered pair to the left of the origin, and to the right of the origin. Choose an x-value one unit to the left of the x-value of the origin, calculate $f(x)$ and then choose an x-value one unit to the right of the x-value of the origin and calculate $f(x)$. The graph will resemble a V, either facing upward or downward, depending on the sign of a.

Example 4.5.1

Create a table of solutions and graph the following absolute-value function:

$f(x) = |x - 4|$

Solution

Comparing this function to the general form for absolute-value functions (shown above), $a = 1$, $h = 4$, $k = 0$. The vertex is (h, k) or $(4, 0)$.

To find two more ordered pairs, choose $x = 3$ and $x = 5$, then compute values of $f(x)$.

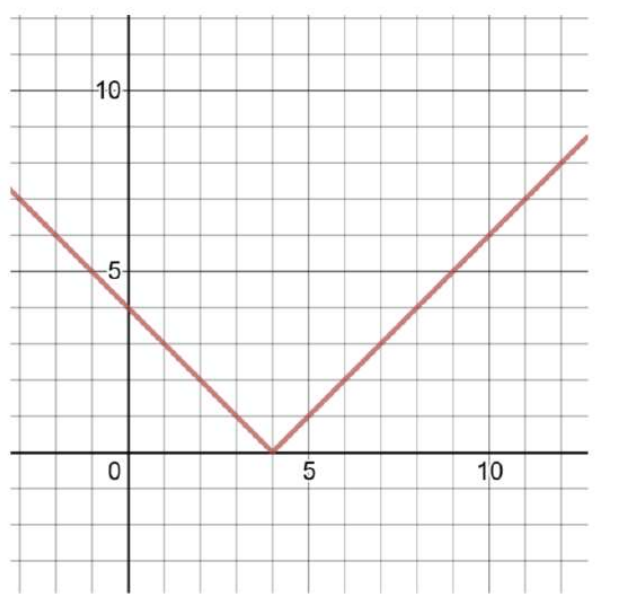

Figure 4.5.1

Table of Solutions for $f(x) = |x - 4|$

x	$f(x)$
3	$f(3) = \|3 - 4\| = \|-1\| = 1$
4	$f(4) = \|4 - 4\| = \|0\| = 0$
5	$f(5) = \|5 - 4\| = \|1\| = 1$

Example 4.5.2

Create a table of solutions and graph the following absolute-value function:

$g(x) = |x + 2| - 5$

Solution

Comparing this function to the general form for absolute-value functions (shown above), $a = 1$, $h = -2$, $k = -5$. The vertex is (h, k)\) or $(-2, -5)$.

To find two more ordered pairs, choose $x = -3$ and $x = -1$, then compute values of $g(x)$

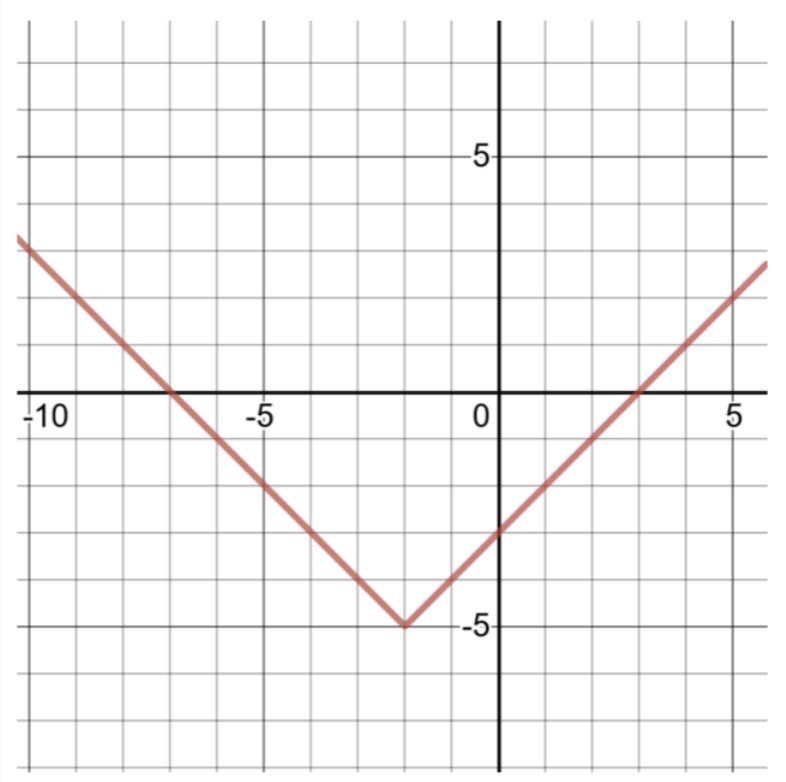

Figure 4.5.2

Table of Solutions for $g(x) = \|x + 2\| - 5$	
x	$g(x)$
-3	$g(-3) = \|-3 + 2\| - 5 = \|-1\| - 5 = 1 - 5 = -4$
-2	$g(-2) = \|-2 + 2\| - 5 = \|0\| - 5 = 0 - 5 = -5$
-1	$g(-1) = \|-1 + 2\| - 5 = \|1\| - 5 = 1 - 5 = -4$

Example 4.5.3

Create a table of solutions and graph the following absolute-value function:

$h(x) = 3|x - 5| + 1$

Solution

Comparing this function to the general form for absolute-value functions (shown above), $a = 3$, $h = 5$, $k = 1$. The vertex is (h, k) or $(5, 1)$.

To find two more ordered pairs, choose $x = 4$ and $x = 6$, then compute values of $h(x)$.

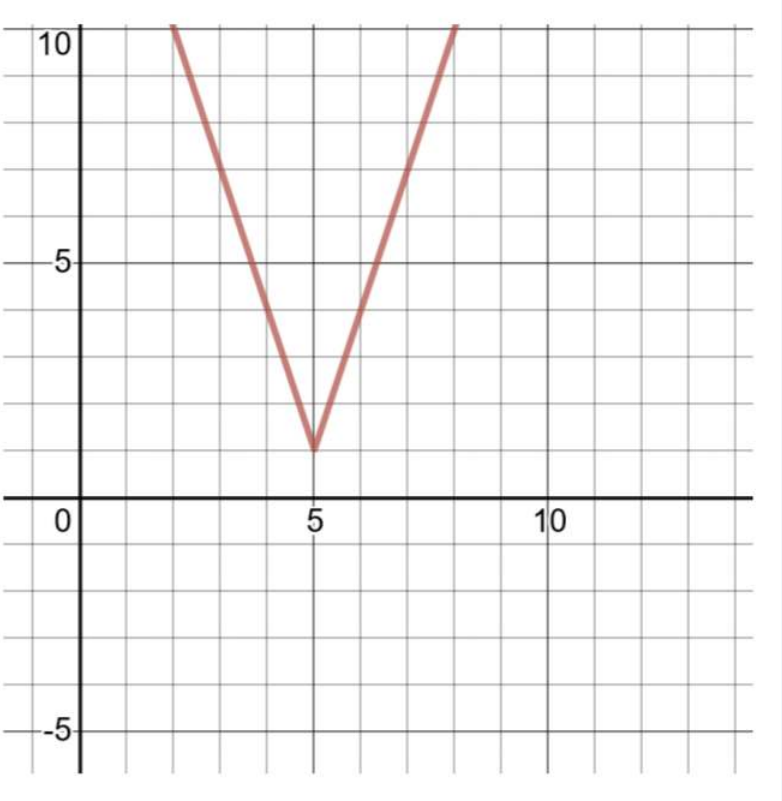

Figure 4.5.3

Table of Solutions for $h(x) = 3\|x-5\|+1$	
x	$h(x)$
4	$g(-3) = \|-3+2\|-5 = \|-1\|-5 = 1-5 = -4$
5	$g(-2) = \|-2+2\|-5 = \|0\|-5 = 0-5 = -5$
6	$g(-1) = \|-1+2\|-5 = \|1\|-5 = 1-5 = -4$

Example 4.5.4

Create a table of solutions and graph the following absolute-value function:

$h(x) = \frac{1}{2}|x-2|+3$

Solution

Comparing this function to the general form for absolute-value functions (shown above), $a = \frac{1}{2}$, $h = 2$, $k = 3$. The vertex is (h, k) or $(2, 3)$.

To find two more ordered pairs, choose $x = 1$ and $x = 3$, then compute values of $h(x)$.

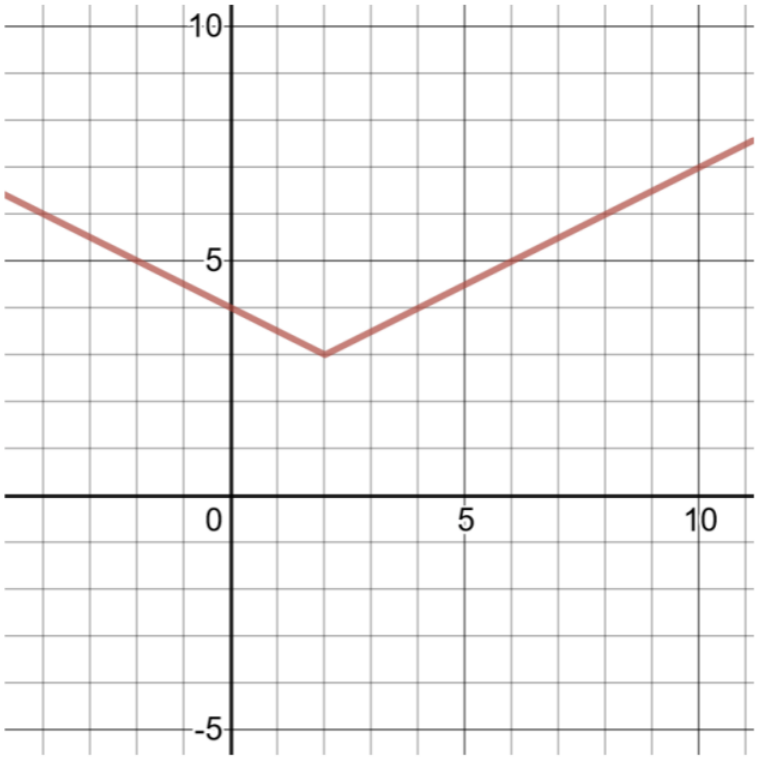

Figure 4.5.4

Table of Solutions for $h(x)=\frac{1}{2}\lvert x-2\rvert+3$	
x	$h(x)$
1	$h(1)=\frac{1}{2}\lvert 1-2\rvert+3=\frac{1}{2}\lvert -1\rvert+3=\frac{1}{2}(1)+3=3\frac{1}{2}$
2	$h(2)=\frac{1}{2}\lvert 2-2\rvert+3=\frac{1}{2}\lvert 0\rvert+3=0+3=3$
3	$h(3)=\frac{1}{2}\lvert 3-2\rvert+3=\frac{1}{2}\lvert 1\rvert+3=\frac{1}{2}(1)+3=3\frac{1}{2}$

Exercise 4.5.1

Create a table of solutions and graph the following absolute-value functions:

1. $f(x)=\lvert x+6\rvert$
2. $g(x)=\frac{1}{3}\lvert x-3\rvert+5$
3. $h(x)=4\lvert x+2\rvert+2$
4. $f(x)=\lvert x-1\rvert-5$

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4.6: Polynomial Functions

A **Polynomial Function** is a function that can be written in the general form:

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0$$

for n a non-negative integer, called the degree of the polynomial. The coefficients a_0, a_1, ..., an are real numbers with leading coefficient an $a_n \neq 0$. The domain of a Polynomial Function is $(-\infty, \infty)$. The graph of a polynomial function of degree n can intersect the x-axis at most n times. These are the roots of the polynomial function.

There are no examples or homework in this section.

4.6.1: Quadratic Functions

Definition: A Function of the Form

$f(x) = ax^2 + bx + c$ where $a \neq 0$

is a Quadratic Function in standard form, and its graph is a parabola. When the leading coefficient, a, is positive, the graph of the Quadratic Function opens upward. When the leading coefficient, a, is negative, the graph of the Quadratic Function opens downward.

Example 4.6.1

Sketch a graph of $f(x) = -x^2 + 5x + 3$ in a rectangular coordinate system. Find the vertex, the x-intercept(s) and the y-intercept algebraically.

Solution

Find the vertex by calculating $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$ with $a = -1$, $b = 5$ and $c = 3$.

$$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) = \quad \text{Find the vertex of the parabola}$$

$$\frac{-5}{2(-1)} =$$

$$\frac{5}{2} = 2.5 \quad \text{Simplify}$$

$$\frac{-5}{2(-1)} = 2.5$$

$$f(2.5) = -(2.5)^2 + 5(2.5) + 3 = 9.25 = \quad f\left(\frac{-b}{2a}\right) = 9.25$$

$$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) = (2.5, 9.25) \quad \text{Vertex of the parabola}$$

To find the intercepts:

$0 = -x^2 + 5x + 3$ x-intercept, set $f(x) = 0$

$0 = -x^2 + 5x + 3$ Use the Quadratic Formula to solve this equation (it can't be factored). Let $a = -1, b = 5$

$x = \frac{-5 \pm \sqrt{(5)^2 - 4(-1)(3)}}{2(-1)}$ Quadratic Formula

$x = \frac{-5 \pm \sqrt{37}}{-2}$ Simplify

$x = -0.54$ or $x = 5.54$ This quadratic function has two roots (x-intercepts).

$f(0) = -0^2 + 5(0) + 3$ y-intercept, set $x = 0$

$f(0) = 3$ y-intercept

Graph the four ordered pairs, and calculate more ordered pairs if needed: $(2.5, 9.25)$, $(-.54, 0)$, $(5.54, 0)$, $(0, 3)$.

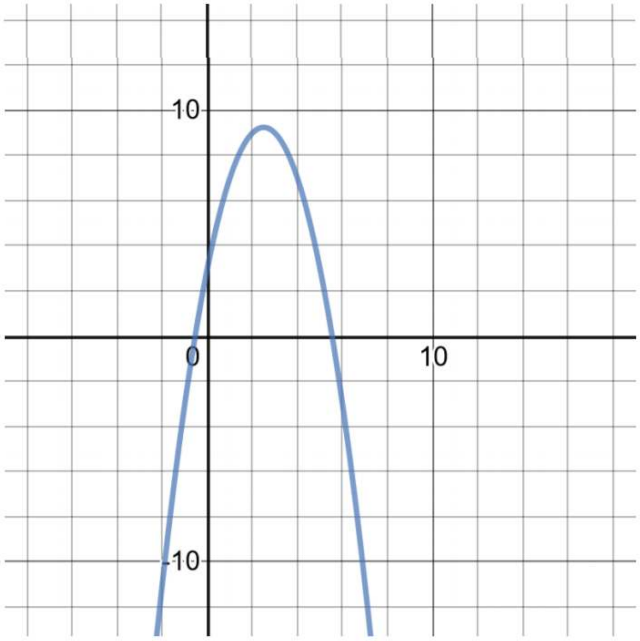

Figure 4.6.1

Exercise 4.6.1

1. $f(x) = 2x^2 - 5x - 5$
2. $f(x) = 0.5x^2 - 6x + 21$
3. $f(x) = -4x^2 - 8x - 3$
4. $f(x) = -4x^2 + 16x - 15$
5. $f(x) = x^2 - 8x + 12$
6. $f(x) = -7x^2 + 100x - 10$

4.6.2: Cubic and Higher Order Functions

Definition: Cubic Function

A Cubic Function is a third-degree polynomial function that can be written in the general form:

$f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$

with 3 as the degree of the cubic function. The coefficients a_0, a_1, a_2, a_3 are real numbers with leading coefficient $a_3 \neq 0$. The domain of a cubic function is $(-\infty, \infty)$.

Example 4.6.2

Factor if possible and graph the function by creating a table of solutions:

$f(x) = x^3 - 4x^2 + 6x - 1$

Solution

This polynomial is of degree 3, and is difficult to factor. Create a table of solutions to graph.

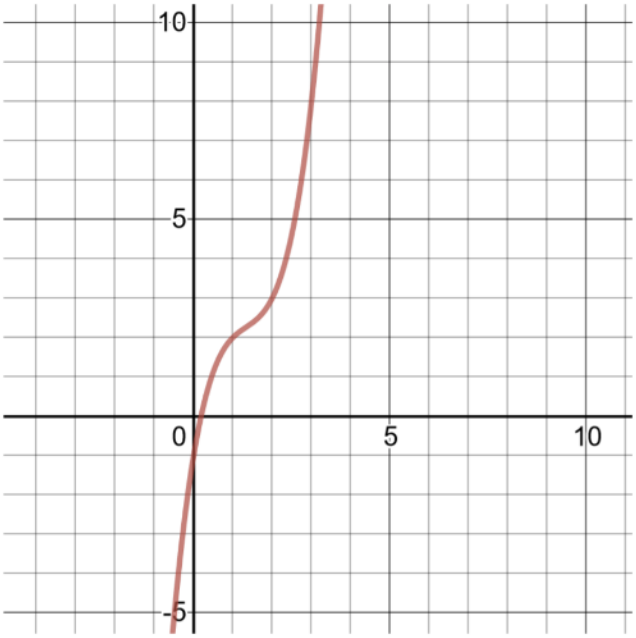

Figure 4.6.2

Table of Solutions for $f(x) = x^3 - 4x^2 + 6x - 1$	
x	$f(x)$
-2	$f(-2) = (-2)^3 - 4(-2)^2 + 6(-2) - 1 = -37$
-1	$f(-1) = (-1)^3 - 4(-1)^2 + 6(-1) - 1 = -12$
0	$f(0) = (0)^3 - 4(0)^2 + 6(0) - 1 = -1$
1	$f(1) = (1)^3 - 4(1)^2 + 6(1) - 1 = 2$
2	$f(2) = (2)^3 - 4(2)^2 + 6(2) - 1 = 3$

Example 4.6.3

Factor if possible and graph the function by creating a table of solutions:

$g(x) = x^4 - 16$

Solution

This polynomial is of degree 4, and because it is a difference of squares, it can be factored into a product of binomials to find the zeroes of the polynomial. Create a table of solutions to graph.

$g(x) = \left(x^2 - 4\right)\left(x^2 + 4\right)$	Factoring into the sum and difference of binomials.
$g(x) = (x-2)(x+2)\left(x^2 + 4\right)$	Further factoring. Set each binomial equal to zero to find the real number zeroes of the polynomial.
$x - 2 = 0, x = 2$	The first real number zero of the polynomial, $(2, 0)$
$x + 2 = 0, x = -2$	The second real number zero of the polynomial, $(2, 0)$
$x^2 + 4 = 0, x^2 = -4$	The third binomial factor does not produce real number zeroes, because no number squared can result in a negative value.

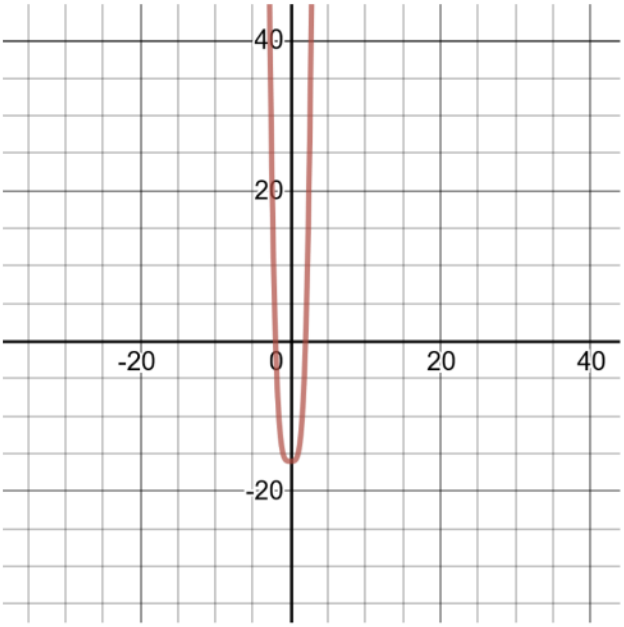

Figure 4.6.3

Table of Solutions for $g(x) = x^4 - 16$	
x	$g(x)$
-2	$g(-2) = (-2)^4 - 16 = 16 - 16 = 0$
-1	$g(-1) = (-1)^4 - 16 = 1 - 16 = -15$
0	$g(0) = (0)^4 - 16 = 0 - 16 = -16$
1	$g(1) = g(1) = (1)^4 - 16 = 1 - 16 = -15$
2	$g(2) = g(2) = (2)^4 - 16 = 16 - 16 = 0$

Example 4.6.4

Factor if possible and graph the function by creating a table of solutions:

$f(x) = x^6 - 5x^2 + 3$

Solution

This polynomial is of degree 6, and is difficult to factor. Create a table of solutions to graph.

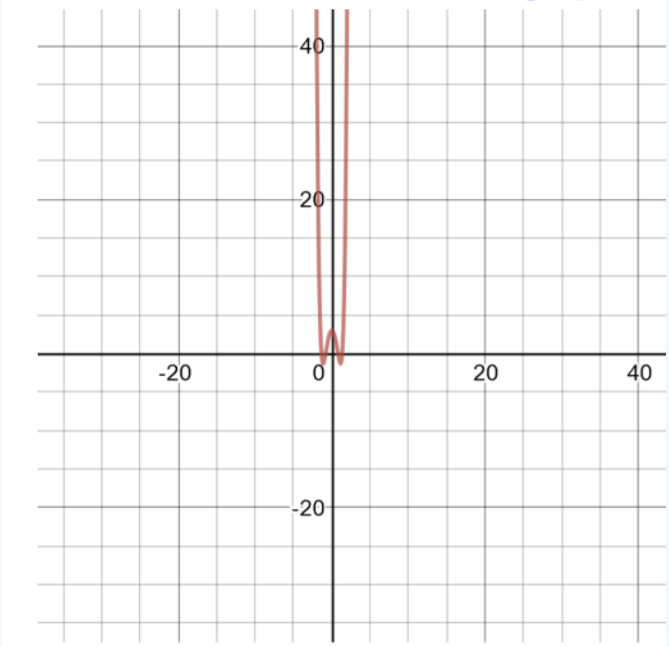

Figure 4.6.4

Table of Solutions for $f(x) = x^6 - 5x^2 + 3$	
x	$f(x)$
-2	$f(-2) = (-2)^6 - 5(-2)^2 + 3 = 47$
-1	$f(-1) = (-1)^6 - 5(-1)^2 + 3 = -1$
0	$f(0) = (0)^6 - 5(0)^2 + 3 = 3$
1	$f(1) = (1)^6 - 5(1)^2 + 3 = -1$
2	$f(2) = (2)^6 - 5(2)^2 + 3 = 47$

? Exercise 4.6.2

1. $f(x) = x^3 - 27$
2. $g(x) = 81x^4 - 16$
3. $h(x) = 2x^5 - 4x^2 - 6x + 3$
4. $f(x) = 5x^6 - 6x^4 + 5$

4.6.3: Rational Functions

Definition: Rational Function

A rational function is a function that can be written as a quotient of polynomials.

$$f(x) = \frac{P(x)}{Q(x)}, Q(x) \neq 0$$

where $P(x)$ and $Q(x)$ are polynomials in one variable x. The domain is the set of all real numbers such that $Q(x) \neq 0$.

Example 4.6.5

For the function, $f(x) = \frac{9}{x-3}$:

a. Graph the function
b. Evaluate the function for $x = 0$ and $x = 2$

Solution

Pay attention to the domain of this function. Division by zero is undefined, so the number(s) that will make the denominator 0 must be excluded from the domain.

In this problem, $x - 3$ is in the denominator of the function. Set $x - 3 = 0$ and solve for x. If $x = 3$ the division is undefined, so exclude the number 3 from the domain of the function. Think of it as always starting with all real numbers $(-\infty, \infty)$ and then removing the values that will cause undefined division.

The domain of this function is $(-\infty, 3) \cup (3, \infty)$.

Rational functions will often have asymptotes, a line that continually approaches a given curve but does not meet it at any finite distance. You will learn about asymptotes in the Curve Sketching section of Math 162.

The graph of this function can be found by making a table of solutions:

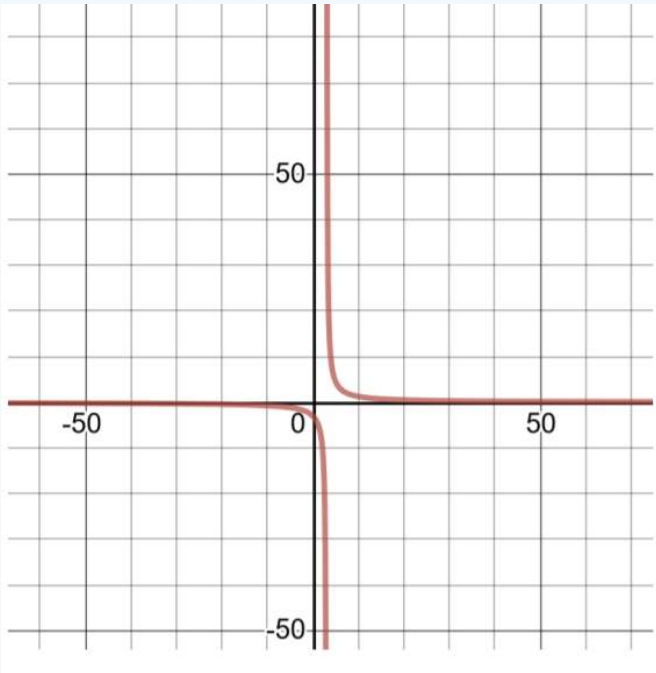

Figure 4.6.5

Table of Solutions for $f(x) = \frac{9}{x-3}$	Domain: $(-\infty, 3) \cup (3, \infty)$
x	$f(x)$
-4	$-\frac{9}{7}$
-3	$-\frac{3}{2}$
-2	$-\frac{9}{5}$
-1	$-\frac{9}{4}$
0	-3
1	$-\frac{9}{2}$
2	-9

Example 4.6.6

For the function, $f(x) = \dfrac{100x}{x^2 - 3x - 4}$

a. Graph the function
b. Evaluate the function for $x = -1$ and $x = 3$

Solution

Pay attention to the domain of this function. Division by zero is undefined, so the number(s) that will make the denominator 0 must be excluded from the domain.

In this problem, $x^2 - 3x - 4$ is in the denominator of the function. Factor the quadratic expression to get $(x-4)(x+1)$ and set each factor equal to zero and solve for x: $x - 4 = 0$, so $x = 4$; $x + 1 = 0$, so $x = -1$. If $x = 4$ or $x = -1$, the division is undefined, so exclude the numbers 4 and −1 from the domain of the function. Think of it as always starting with all real numbers $(-\infty, \infty)$ and then removing the values that will result in undefined division.

The domain of this function is $(-\infty, -1) \cup (-1, 4) \cup (4, \infty)$. The graph of this function can be found by making a table of solutions:

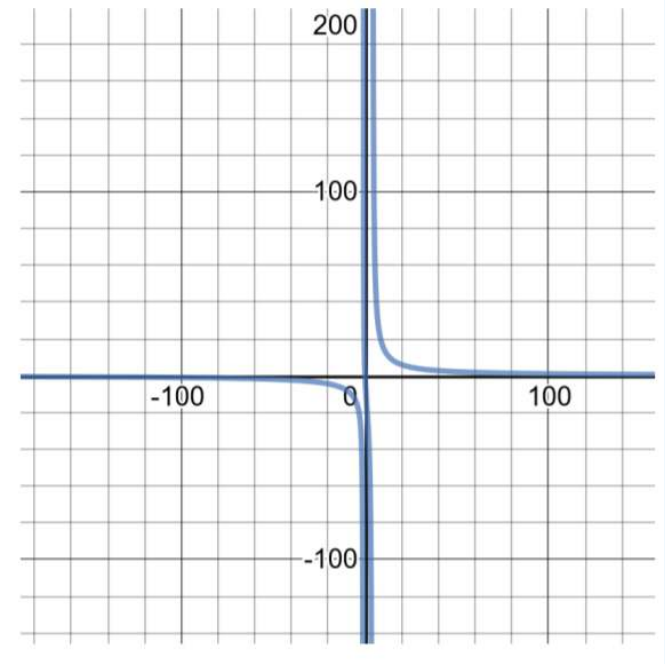

Figure 4.6.6

Table of Solutions for $f(x) = \dfrac{100x}{x^2 - 3x - 4}$	Domain: $(-\infty, -1) \cup (-1, 4) \cup (4, \infty)$
x	$f(x)$
-4	−16.667
-3	−21.429
-2	−33.333
-1	undefined
0	0
1	−16.667
2	−33.333
3	-75

4	undefined

Exercise 4.6.3

1. $f(x) = \dfrac{3x+6}{x-1}$
2. $f(x) = \dfrac{9}{x^2-9}$
3. $f(x) = \dfrac{x^2-4}{x^2-4x}$

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4.7: Domain and Range of a Function

Definition: Domain and Range of a Function

The domain of a function is all possible values of x that can be used as input to the function, which will result in a real number as the output. The range of a function is the set of all possible output values of a function.

Example 4.7.1

Find the domain and range of the following function:

$f(x) = 5x + 3$

Solution

Any real number, negative, positive or zero can be replaced with x in the given function. Therefore, the domain of the function $f(x) = 5x + 3$ is all real numbers, or as written in interval notation, is: $D : (-\infty, \infty)$. Because the function $f(x) = 5x + 3$ is a polynomial of degree 1, it is a straight line (without any breaks or holes).

The range of any polynomial of degree 1 is all real numbers or written in interval notation, is: $R : (-\infty, \infty)$.

Example 4.7.2

Find the domain and range of the following function:

$g(x) = 2\sqrt{x-4}$

Solution

Pay attention to the square root part of this function. The radicand (what's inside the square root) must be nonnegative. Set the radicand greater than or equal to zero to find the domain:

$$\begin{aligned} x - 4 &\geq 0 && \text{Set the radicand greater than or equal to 0} \\ x &\geq 4 && \text{Solve the inequality} \\ D &: [4, \infty) && \text{Write the solution in interval notation} \end{aligned}$$

Therefore, the domain of the function $g(x) = 2\sqrt{x-4}$ is all real numbers in the interval from $[4, \infty)$, which is written $D : [4, \infty)$.

To find the range of $g(x) = 2\sqrt{x-4}$, let's observe the behavior of the function for different values of x that are in the domain.

Let $x = 4$, $g(4) = 2\sqrt{4-4}$, so $g(4) = 0$.

Let $x = 5$, $g(5) = 2\sqrt{5-4}$, so $g(5) = 2$.

Let $x = 8$, $g(8) = 2\sqrt{8-4}$, so $g(8) = 4$.

Any non-negative value chosen for x will result in a non-negative value for $g(x)$. The function values for the range (the output from the function $g(x)$) are non-negative numbers, written as $R : [0, \infty)$.

Example 4.7.3

Find the domain and range of the following function:

$h(x) = -2x^2 + 4x - 9$

Solution

Any real number, negative, positive or zero can replace x in the given function.

Therefore, the domain of the function $h(x) = 2x^2 + 4x - 9$ is all real numbers, or as written in interval notation, is: $D : (-\infty, \infty)$.

Because the function $h(x) = 2x^2 + 4x - 9$ is a quadratic of degree 2, when graphed, it is a parabola (without any breaks or holes). Identify two things about this parabola:

1. Which way does it open, up or down? and

2. Where is the vertex?

The sign of the coefficient of the leading term of the quadratic function ($2x^2$) shows which way the parabola opens. The coefficient is 2, and since it's positive, the quadratic function opens upward.

Now find the vertex. The y-value of the vertex ordered pair will show where the range begins.

The vertex is $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$, with $a = 2$ and $b = 4$.

The vertex is $\left(-\frac{4}{2*2}, f\left(-\frac{4}{2*2}\right)\right)$

The vertex is $(-1, f(-1))$, which is $(-1, 2*(-1)^2 - 9))$ or $(-1, -11)$

The range will start at −11, and continue to increase, since the parabola opens upward. $R : [-11, \infty)$

Example 4.7.4

Find the domain and range of the following function:

$j(x) = |z - 6| - 3$

Solution

This function contains an absolute value. Any value can be chosen for z, so the domain of the function is all real numbers, or as written in interval notation, is: $D : (-\infty, \infty)$

To find the range, examine inside the absolute value symbols. This quantity, $|z - 6|$ will always be either 0 or a positive number, for any values of z. First, find what makes the expression z−6 equal to zero, which is the number 6.

$j(x) = \|z - 6\| - 3$	Original function
$j(x) = \|6 - 6\| - 3$	Replace z with 6
$j(x) = \|0\| - 3$	Simplify
$j(x) = -3$	$j(x)$ is -3

Therefore, the range of the function $j(x) = |z - 6| - 3$ is −3 or higher, or as written in interval notation, is: $R : [-3, \infty)$

Some types of functions are more difficult to work with. Here are some examples of functions where the domain can be found but the range would be too difficult to find, and out of the scope of this course:

Example 4.7.5

Find the domain of the following functions:

$$f(x) = \frac{x - 4}{x^2 - 2x - 15}$$

Solution

With any rational function (a quotient of polynomials), be aware of division by 0. Set the denominator polynomial equal to 0 and solve.

$x^2 - 2x - 15 = 0$	Set the denominator function equal to 0
$(x - 5)(x + 3) = 0$	Factor the quadratic equation
$x - 5 = 0$	Set the first binomial factor equal to zero
$x = 5$	Solve the first binomial factor
$x + 3 = 0$	Set the second binomial factor equal to zero
$x = -3$	Solve the second binomial factor

There are two solutions to the quadratic equation, 5 and −3.

These values must be excluded from the domain, because if x is either 5 or −3, the denominator will equal zero.

Division by zero is undefined. The domain of the function $f(x) = x - 4x^2 - 2x - 15$ is $(-\infty, -3) \cup (-3, -5) \cup (-5, \infty)$.

Example 4.7.6

Find the domain of the following function:

$g(x) = \dfrac{x}{x^2 - 9}$

Solution

Once again this is a rational function, and the concern is to avoid division by 0. Set the denominator function equal to 0 and solve.

$$\begin{aligned} x^2 - 9 &= 0 && \text{Set the denominator function equal to 0} \\ (x-3)(x+3) &= 0 && \text{Factor the quadratic equation} \\ x - 3 &= 0 && \text{Set the first binomial factor equal to zero} \\ x &= 3 && \text{Solve the first binomial factor} \\ x + 3 &= 0 && \text{Set the second binomial factor equal to zero} \\ x &= -3 && \text{Solve the second binomial factor} \end{aligned}$$

There are two solutions to the quadratic equation, 3 and −3. These values must be excluded from the domain, because if x is either 3 or −3, the denominator will equal zero. Division by zero is undefined. The domain of the function $g(x) = \dfrac{x}{x^2 - 9}$ is $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$.

Example 4.7.7

Find the domain of the following function:

$g(t) = \sqrt{6 + t - t^2}$

Solution

The radicand of this square root function must be non-negative. Set the radicand greater than or equal to 0 and solve.

$$\begin{aligned} 6 + t - t^2 &\geq 0 && \text{Set the radicand equal to 0} \\ -t^2 + t + 6 &\geq 0 && \text{Rewrite the function with the leading term first} \\ (-t+3)(t+2) &= 0 && \text{Factor the quadratic equation} \\ -t + 3 &= 0 && \text{Set the first binomial factor equal to zero} \\ t &= 3 && \text{Solve the first binomial factor} \\ t + 2 &= 0 && \text{Set the second binomial factor equal to zero} \\ t &= -2 && \text{Solve the second binomial factor} \end{aligned}$$

There are two values that will make the radicand of this square root function zero, 3 and −2.

Since the radicand must be non-negative, test the regions between the found solutions.

If $x < -2$, for example, −4, $g(-4) = \sqrt{6 + (-4) - (-4)^2}$ is negative, which is not allowed for the radicand.

If x is between −2 and 3, for example, 0, $g(0) = \sqrt{6 + (0) - (0)^2}$ is positive. This region between −2 and 3 will be in the domain of the function.

There is one more region to check, where $x > 3$. Let $x = 4$. $g(4) = \sqrt{6 + (4) - (4)^2}$ is negative, which is not allowed for the radicand. The domain of the function $g(t) = \sqrt{6 + t - t^2}$ is $[-2, 3]$

Exercise 4.7.1

Find the domain and range of the following functions:

1. $f(x) = x^2 - 8x + 12$
2. $g(x) = \sqrt{x + 10}$
3. $h(x) = |-2x + 1|$

Exercise 4.7.2

Find the domain of the following functions:

1. $f(x) = \dfrac{6x + 7}{5x + 2}$

2. $f(x) = \dfrac{2x}{2x^2 + 3x - 20}$
3. $f(x) = \dfrac{4x + 11}{x^2 + 6x + 9}$
4. $f(x) = \dfrac{3x}{x^2 - 5x - 14}$
5. $f(x) = \dfrac{2x + 1}{6x^2 - x - 2}$
6. $f(x) = \dfrac{-6}{25x^2 - 4}$

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4.8: Graphing Functions (without using Calculus)

There are some basic functions, called toolkit functions, that students should recognize by their function definition and their graph. For each of these functions, x is the input variable, and $f(x)$ is the output variable. The following graphics are from the OER textbook Business Calculus by Calaway, Hoffman and Lippman, 2013 and are used with permission (Creative Commons Attribution 3.0 United States License).

Toolkit Functions

Linear

Constant:	$f(x) = c$, where c is a constant (number)
Identity:	$f(x) = x$
Absolute Value:	$f(x) = \lvert x \rvert$

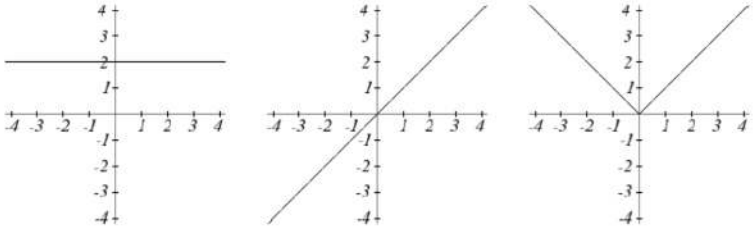
Power

Quadratic:	$f(x) = x^2$
Cubic:	$f(x) = x^3$
Reciprocal:	$f(x) = \frac{1}{x}$
Reciprocal squared:	$f(x) = \frac{1}{x^2}$
Square root:	$f(x) = \sqrt[3]{x} = \sqrt{x}$
Cube root:	$f(x) = \sqrt[3]{x}$

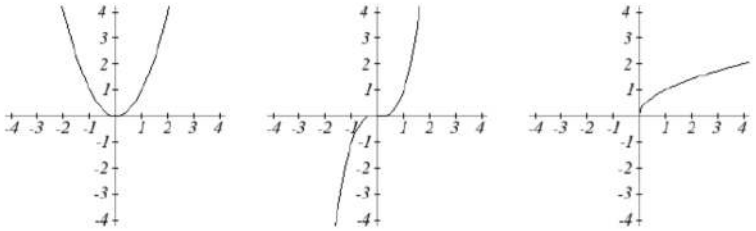
Figure 4.8.1

Graphs of the Toolkit Functions

Constant Function: $f(x) = 2$ Identity: $f(x) = x$ Absolute Value: $f(x) = |x|$



Quadratic: $f(x) = x^2$ Cubic: $f(x) = x^3$ Square root: $f(x) = \sqrt{x}$



Cube root: $f(x) = \sqrt[3]{x}$ Reciprocal: $f(x) = \frac{1}{x}$ Reciprocal squared: $f(x) = \frac{1}{x^2}$

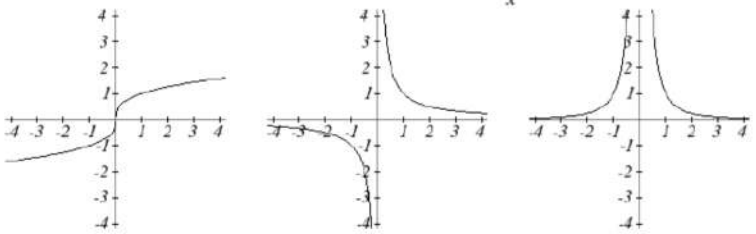


Figure 4.8.2

Unlike a traditional STEM Calculus I course, this Calculus for Business and Social Sciences course does not teach graphing functions by using function transformations.

Students in this class are expected to make a table of solutions and graph the function. Students will also learn how to graph functions using Calculus!

Example 4.8.1

Graph the following functions:

a. $f(x) = \sqrt[3]{x}$

b. $f(x) = \dfrac{1}{x^2 - 3}$

Solution

a. Make a table of solutions and define the domain of the function.

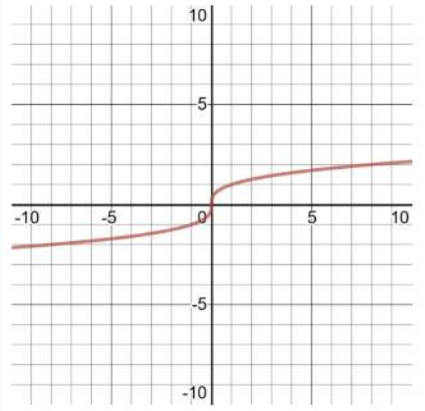


Figure 4.8.3

Table of Solutions for $f(x) = \sqrt[3]{x}$ Domain $(-\infty, \infty)$	
x	$f(x)$
-8	-2
-1	-1
0	0
1	1
8	2

b. Make a table of solutions and define the domain of the function. To identify the domain of this rational function, pay attention to the denominator. The denominator cannot equal 0. Set the denominator = 0 to solve for x and find the values that will not be allowed for x.

$$\begin{aligned} f(x) &= \frac{1}{x^2 - 3} \\ 0 &= x^2 - 3 \\ 3 &= x^2 \\ \pm\sqrt{3} &= \sqrt{x^2} \\ \pm\sqrt{3} &= x \end{aligned}$$

These numbers need to be excluded from the domain of this function $-\sqrt{3}$ (about −1.732)) and $\sqrt{3}$ (about 1.732)).

To correctly graph this function, it's important to examine the behavior around these numbers that are excluded from the domain. Thus, the reason for so many ordered pairs in the table of solutions. Think of it like this: start with a domain of $(-\infty, \infty)$, but must remove any numbers that will cause problems (like in this case, numbers that will cause the denominator of the function to be 0, because division by 0 is undefined). Graphing functions like this by hand is very tedious, but it's an important skill for a student to have to succeed in Calculus for Business and Social Sciences.

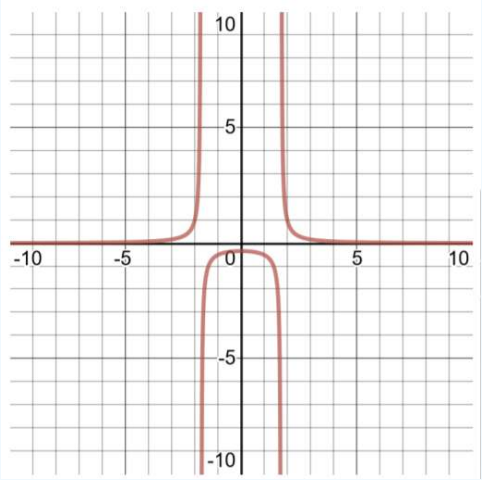


Figure 4.8.4

Table of Solutions for $f(x) = \sqrt[3]{x}$ Domain $(-\infty, \infty)$	
x	$f(x)$
-4	0.077
-3	0.167
-2	1
-1.5	-1.333
-1	-0.5
-0.5	-0.364
0	0
0.5	-0.364
1	-0.5
1.5	-1.333
2	1
3	0.167
4	0.077

Practice Problems: Graph the following functions, paying attention to the domain of the function.

1. $f(x) = 2x^3$
2. $f(x) = \dfrac{1}{2x^2}$
3. $f(x) = 4|x - 2|$
4. $f(x) = \dfrac{1}{3}x - 12$
5. $f(x) = \dfrac{1}{x - 7}$
6. $f(x) = 3\sqrt{2x^3 + 1}$

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4.9: Function Composition

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Composition of Functions

When the output of one function is used as the input of another, we call the entire operation a **composition of functions**. We write $f(g(x))$, and read this as "f of g of x" or "f composed with g at x".

An alternate notation for composition uses the composition operator: $\circ$

$(f \circ g)(x)$ is read "f of g of x" or "f composed with g at x", just like $f(g(x))$.

Figure 4.9.1

The notation $f(g(x))$ and $g(f(x))$ may be easier to understand than using the composition operator. For $f(g(x))$, think of wrapping a package. The gift is put into the box (the gift is $g(x)$, the box is $f(x)$) and the wrapped present, $f(x)$, contains the gift $g(x)$.

Example 4.9.1

If $f(x) = x^2 - 2$ and $g(x) = \sqrt{x}$, find:

a. $f(g(x))$ and the domain of the composite function
b. $g(f(x))$ and the domain of the composite function

Solution

a. The composition of functions, $f(g(x))$ is:

$f(g(x))$	Function composition, f of g of x
$f(\sqrt{x})$	Replace $g(x)$ with $\sqrt{x}$
$(\sqrt{x})^2 - 2$	In the function $f(x)$, every x is replaced with $g(x) = \sqrt{x}$
$x - 2$	$f(g(x))$, answer simplified.

The domain of the composite function contains the restrictions of the domain of the inner function, as well as the restrictions of the composite function.

The domain of the inner function, $g(x) = \sqrt{x}$ is that x must be nonnegative, or in interval notation $[0, \infty)$

The domain of the composite function, $x - 2$ is all real numbers, $(-\infty, \infty)$

Therefore, the domain of $f(g(x))$ is $[0, \infty)$.

b. The composition of functions, $g(f(x))$ is:

$g(f(x))$	Function composition, g of f of x
$g(x^2 - 2)$	Replace $f(x)$ with $x^2 - 2$
$\sqrt{x^2 - 2}$	In the function $g(x)$, every x is replaced with $f(x) = x^2 - 2$
$x^2 - 2$	$g(f(x))$, answer simplified.

The domain of the composite function contains the restrictions of the domain of the inner function, as well as the composite function.

The domain of the inner function, $f(x) = x^2 - 2$ is all real numbers, or in interval notation $(-\infty, \infty)$

The domain of the composite function, $\sqrt{x^2 - 2}$ is that the quantity $x^2 - 2$ must be nonnegative, or $x^2 - 2 \geq 0$.

Solving $x^2 - 2 \geq 0$ for x, $x \geq 2$ and $x \leq -2$. In interval notation, $(-\infty, -2] \cup [2, \infty)$

Therefore, the domain of the composite function, g(f(x)) is the more restrictive domain, $(-\infty, -2] \cup [2, \infty)$.

Example 4.9.2

If $f(x)=\dfrac{1}{x-4}$ and $g(x)=\dfrac{5}{x}+4$, find:

a. $f(g(x))$ and the domain of the composite function
b. $g(f(x))$ and the domain of the composite function

Solution

a. The composition of functions, $f(g(x))$ is:

$f(g(x))$	Function composition, f of g of x
$f\left(\dfrac{5}{x}+4\right)$	Replace $g(x)$ with $\dfrac{5}{x}+4$
$\dfrac{1}{(5x+4)-4}$	In the function $f(x)$, every x is replaced with $g(x)=\dfrac{5}{x}+4$
$\dfrac{1}{\dfrac{5}{x}}$	Simplify
$\dfrac{x}{5}$	$f(g(x))$, answer simplified.

The domain of the composite function contains the restrictions of the domain of the inner function, as well as the restrictions of the composite function.

The domain of the inner function, $g(x)=5x+4$ is all values of x such that x must not be 0, or in interval notation $(-\infty,0)\cup(0,\infty)$

The domain of the composite function, $\dfrac{x}{5}$ is all real numbers, $(-\infty,\infty)$ Therefore, the domain of $f(g(x))$ is $(-\infty,0)\cup(0,\infty)$

b. The composition of functions, $g(f(x))$ is

$g(f(x))$	Function composition, g of f of x
$g\left(\dfrac{1}{x-4}\right)$	Replace $f(x)$ with $\dfrac{1}{x-4}$
$\dfrac{5}{\dfrac{1}{x-4}}+4$	In the function $g(x)$, every x is replaced with $f(x)=\dfrac{1}{x-4}$
$5(x-4)+4$	Simplify the fraction
$5x-20+4$	Simplify more
$5x-16$	$g(f(x))$, answer simplified.

The domain of the composite function contains the restrictions of the domain of the inner function, as well as the composite function.

The domain of the inner function, $f(x)=\dfrac{1}{x-4}$ is that $x\neq 4$, or in interval notation $(-\infty,4)\cup(4,\infty)$

The domain of the composite function, $5x-16$ is all real numbers, $(-\infty,\infty)$.

Therefore, the domain of the composite function, $g(f(x))$ is the more restrictive domain, $(-\infty,4)\cup(4,\infty)$.

? Exercise 4.9.1

For the given functions, find both $f(g(x))$ and $g(f(x))$, and find the domain of the composite function.

1. $f(x)=3x^2+x-10$, $g(x)=1-20x$
2. $f(x)=3x-2$, $g(x)=\dfrac{1}{3}x+\dfrac{2}{3}$
3. $f(x)=4x-1$, $g(x)=\sqrt{6+7x}$
4. $f(x)=5x+2$, $g(x)=x^2-14x$
5. $f(x)=x^2-2x+1$, $g(x)=8-3x^2$
6. $f(x)=x^2+3$, $g(x)=\sqrt{5+x^2}$

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4.10: Finding all Real Roots of a Function

To find the real roots of a function, find where the function intersects the x-axis. To find where the function intersects the x-axis, set $f(x) = 0$ and solve the equation for x.

If the function is a linear function of degree 1, $f(x) = mx + b$ and the x-intercept is the root of the equation, found by solving the equation for x. To find the roots of quadratic equations, there are several ways to find the zeros:

1. Fully factor the quadratic expression.
2. Use the quadratic formula, with the quadratic equation in the form $Ax^2 + Bx + C = 0$.
3. Complete the square on the quadratic expression (not included in this workbook).

Some cubic equations can also be solved easily, if the polynomial can be factored to find the zeroes. Also, the cubic equation can be factored if written in the form of a sum or difference of perfect cubes. If they are not in this form, then a calculator or a computer could find the roots of a cubic equation.

The focus of our class is to work with polynomials whose roots can be found using traditional algebraic techniques. For details about how to factor an expression, please refer to the section Factoring/Finding Polynomial Solutions (zeros). For details about how to use the Quadratic Formula, please refer to that section in the document.

? Exercise 4.10.1

Find the real roots of each equation by factoring or using the Quadratic Formula. Express the exact final simplified answers (real numbers or simplified radical expressions).

1. $x^2 + x - 12 = 0$
2. $-6x^2 + x + 12 = 0$
3. $4x^2 + 5x - 6 = 0$
4. $\frac{1}{2}a^2 + a - 12 = 0$
5. $2x^2 + 7x - 15 = 0$
6. $12x^2 - 9x - 3 = 0$

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4.11: Piecewise-Definition Functions

Definition: Piecewise-Defined Functions

Piecewise-Defined Functions are functions that are defined using different equations for different parts of the domain.

Example 4.11.1

Evaluate the following piecewise-defined function for the given values of x, and graph the function:

$$f(x) = \begin{cases} -2x+1 & -1 \le x < 0 \\ x^2+2 & 0 \le x \le 2 \end{cases}$$

Solution

To graph this function, make a table of solutions:

Table of Solutions for $f(x) = -2x+1$ Domain $-1 \le x < 0$	
x	$f(x)$
-1	3
0	1 (open circle here, 0 not in the domain)

Table of Solutions for $f(x) = x^2+2$ Domain $0 \le x \le 2$	
x	$f(x)$
0	2
1	3
2	6

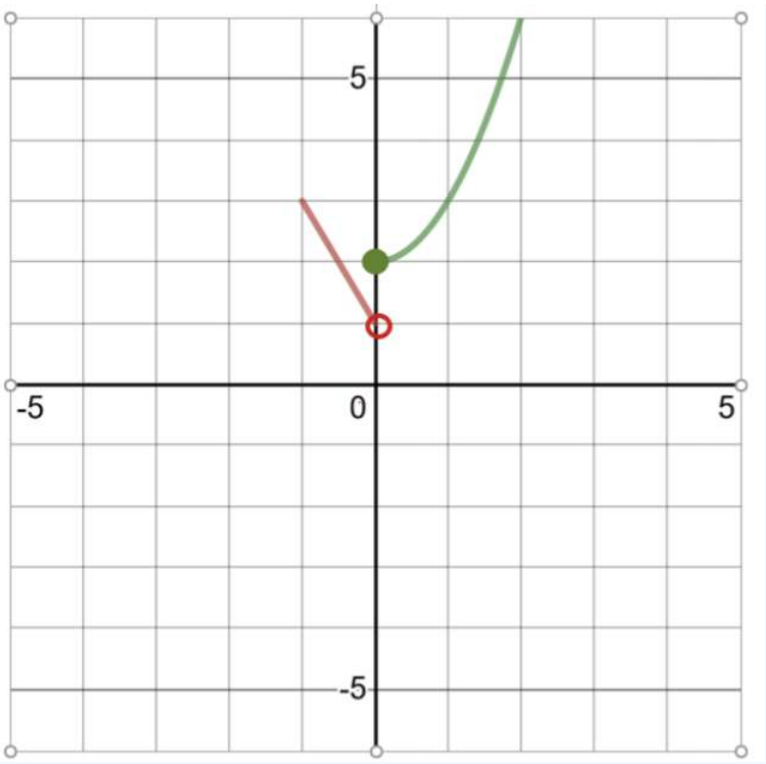

Figure 4.11.1

Example 4.11.2

Evaluate the following piecewise-defined function for the given values of x, and graph the function:

$$f(x) = \begin{cases} -x+1 & x \leq -1 \\ 2 & -1 < x \leq 1 \\ -x+3 & x > 1 \end{cases}$$

Solution

To graph this function, once again make a table of solutions:

Table of Solutions for $f(x) = -x+1$ Domain $x \leq -1$	
x	$f(x)$
-3	4
-2	3
-1	2 (closed circle here, -1 is in the domain)

Table of Solutions for $f(x) = 2$ Domain $-1 < x \leq 1$	
x	$f(x)$
-1	2 (open circle filled in by the previous function, -1 not in the domain)
0	2
1	2 (closed circle here, 1 is in the domain)

Table of Solutions for $f(x) = -x + 3$ Domain $x > 1$	
x	$f(x)$
1	2 (open circle filled in by the previous function, 1 not in the domain)
2	1
3	0

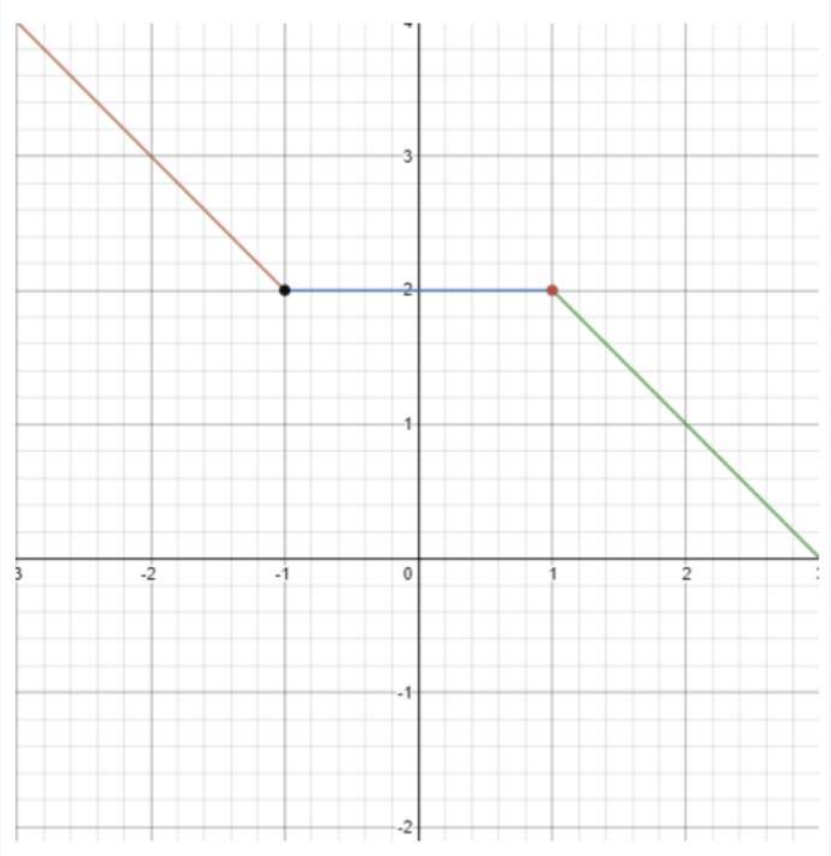

Figure 4.11.2

? Exercise 4.11.1

Evaluate the following piecewise-defined functions for the given values of x, and graph the functions:.

1. $f(x) = \begin{cases} x & x < 0 \\ 2x + 1 & x \geq 0 \end{cases}$
2. $g(x) = \begin{cases} 4 - x & x < 2 \\ 2x - 2 & x \geq 2 \end{cases}$
3. $h(x) = \begin{cases} -x - 1 & x < -1 \\ 0 & -1 \leq x \leq 1 \\ x + 1 & x > 1 \end{cases}$
4. $g(x) = \begin{cases} 6 & -8 \leq x < -4 \\ 3 & -4 \leq x \leq 5 \end{cases}$
5. $f(x) = \begin{cases} -x + 1 & -1 \leq x < 1 \\ \sqrt{x - 1} & 1 \leq x \leq 5 \end{cases}$

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4.12: Applied Examples of Functions

Applied examples of function (AKA word problems!) can take many forms.

Be prepared to use any type of function that has been reviewed in this chapter.

Example 4.12.1

A summer job pays time and a half for overtime if a worker works more than 40 hours. After 40 hours, the worker is paid 1.5 times the hourly rate of $\$12.00$ per hour.

A. Write and graph a piece-wise function that gives the weekly pay P in terms of the number of hours worked h. HINT: This can be tricky, think about how to express the number of hours above 40.
B. How much will be earned working 45 hours?

Solution

A. To write the piece-wise function:

$$P(h) = \begin{cases} 12h & 0 < h \leq 40 \\ 12(40) + 1.5(12)(h-40) & h > 40 \end{cases}$$

To graph this function, make a table of solutions:

Table of Solutions for $P(h) = 12h$ Domain $0 < h \leq 40$	
h	$P(h)$
1	$\$12.00$
10	$\$120.00$
40	$\$480.00$

Table of Solutions for $P(h) = 12(40) + 1.5(7)(h-40)$ Simplified $P(h) = 18h - 240$ Domain $h > 40$	
h	$P(h)$
41	498
45	570
50	660
60	840

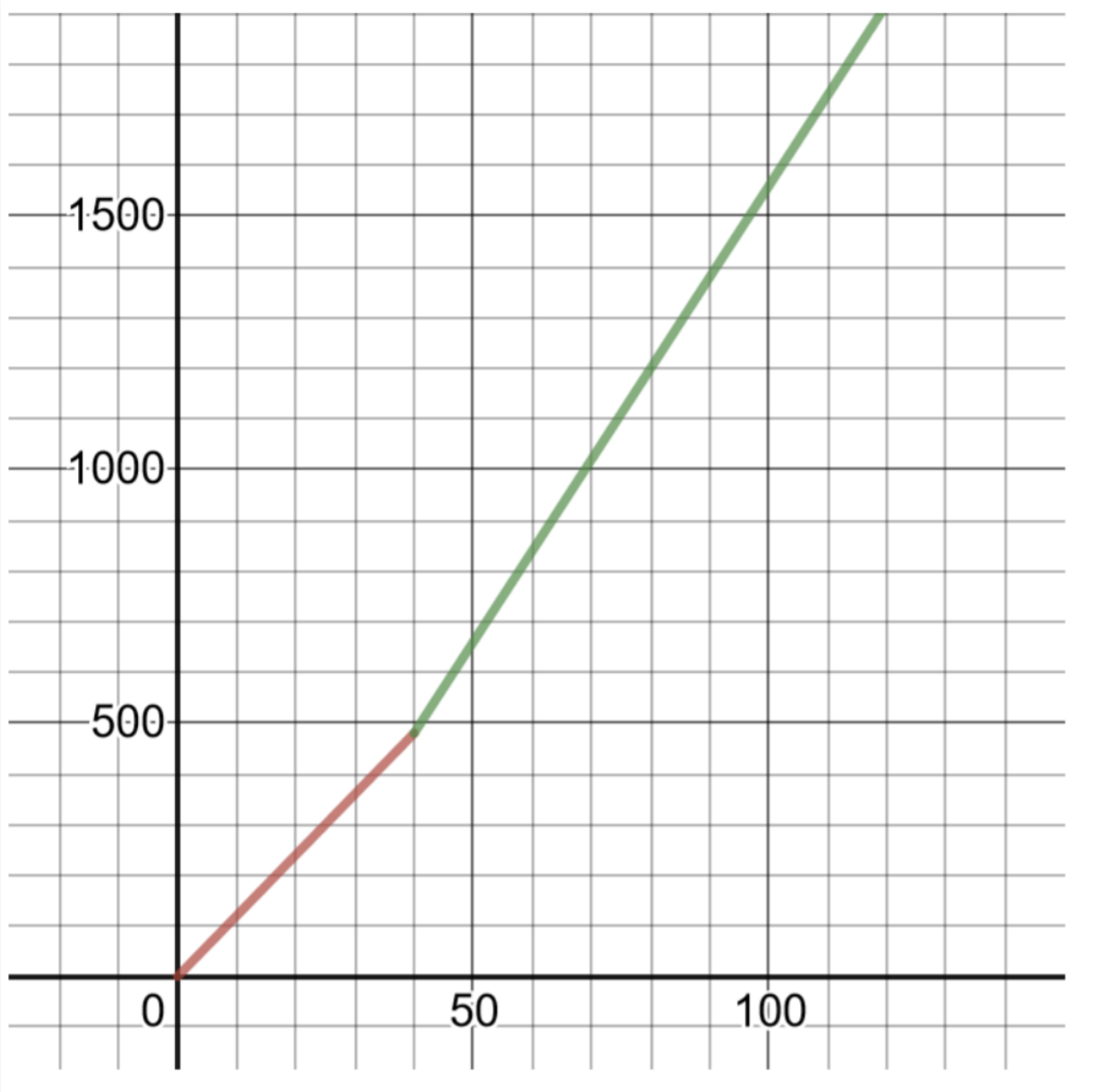

Figure 4.12.1

B. $570.00 for 45 hours of work (see Table of Solutions)

Example 4.12.2

NASA launches a rocket at $t = 0$ seconds. Its height, in meters above sea-level, as a function of time is given by $h(t) = -4.9t^2 + 46t + 227$.

A. Assuming that the rocket will splash down into the ocean, at what time does splashdown occur?
B. How high above sea-level does the rocket get at its peak?

Solution

A. At splash down, $h(t) = 0$, so set the function equal to 0 and solve for t.

$0 = -4.9t2 + 46t + 227$

Use the Quadratic Formula to solve this equation, with $a = -4.9$, $b = 46$, $c = 227$

$t = \frac{-46 \pm \sqrt{46^2 - 4(-4.9)(227)}}{2(-4.9)}$	Quadratic Formula
$t = \frac{-46 \pm \sqrt{2116 + 4449.2}}{-9.8}$	Simplify the radical
$t = \frac{46 \pm \sqrt{6565.2}}{9.8}$	Further simplify the radical, divide all terms by -1 (still have $\pm$)
$t = \frac{46 \pm 81.026}{9.8}$	Square root
$t = \frac{46 + 81.026}{9.8}$	Addition
$t = \frac{46 - 81.026}{9.8}$	Subtraction
$t = 12.96$ and $t = -3.57$	Two solutions, reject negative solution because time cannot be negative
$t = 12.96$ seconds	Final Answer

B. How high above sea-level does the rocket get at its peak?

The sign of the coefficient of the leading term of the quadratic function $h(t) = -4.9t^2 + 46t + 227$ shows which way the parabola opens. The coefficient is -4.9, and since it's negative, the quadratic function opens downward.

Now we need to find the vertex. The y-value of the vertex ordered pair will show where the range begins.

The vertex is $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$, with $a = -4.9$ and $b = 46$

The vertex is $\left(-\frac{46}{2(-4.9)}, f\left(-\frac{46}{2(-4.9)}\right)\right)$

The vertex is $(4.694, f(4.694))$ which is $(4.694, (-4.9)(4.694)^2 + (46)(4.694) + 227))$ or $(4.694, 334.959)$

The height of the rocket at its peak is 334.959 meters above sea level.

Example 4.12.3

To promote group sales for an Alaskan cruise, a travel agency reduces the regular ticket price of $\$4500$ by $\$5$ for each person in the group.

A. Write a linear equation that gives the per-person cost $f(p)$, if p people travel in the group.
B. Use the equation to determine the cost for 50 people.

Solution

A. Since the per-person cost is reduced the same amount for each person, this is a linear equation.

Use $f(x) = mx + b$, or let's write it as $f(p) = mp + b$, with $f(p)$ the per-person cost.

$f(p) = mp + b$

Since the per-person cost is reduced by $5 for each person in the group, that is the slope of the line.

$f(p) = -5p + b$	Slope-intercept form of the equation of a line
$f(p) = -5p + 4500$	The y-intercept is the starting point, so the regular ticket price of $\$4500 is they - intercept$
$f(p) = -5p + 4500$	Linear Equation

B. Use the equation to determine the cost for 50 people.

$f(50) = -5(50) + 4500$	Replace p with 50 people in the Linear Equation
$f(50) = -250 + 4500$	Simplify
$f(50) = 4250$	Simplify
If 50 people take the cruise, the cost per-person for the cruise is $4250	Final Answer

Exercise 4.12.1

Complete the following applied function problems:

1. A tailor charges a basic fee of $\$20$ plus $\$5$ per letter to sew an athlete's name on the back of a jacket.
   A. Write a linear equation that will find the cost to have a name containing x letters sewn on the back of a jacket.

B. What is the cost for a name with 11 letters?

2. In Acapulco, Mexico, a popular tourist attraction is to watch men diving from a cliff to the water 75 feet below. A diver's height $f(t)$ above the water (in feet), t seconds after diving, is given by $f(t) = -16t^2 + 75$.
   - How long does a dive last?
3. In a certain country, income tax is assessed as follows: There is no tax on income up to $\$15000$. Income over $\$15000$ and up to $\$30000$ is taxed at 12. Income over $\$30000$ is taxed at 18.
   A. Write the piecewise function of the above information and sketch its graph.
   B. Determine the amount of tax to be paid on an income of $\$18000$.

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CHAPTER OVERVIEW

5: Exponents and Exponent Rules

5.1: Definition of a^n
5.2: The Product Rule for Exponents
5.3: The Quotient Rule of Exponents
5.4: Zero Exponent Rule
5.5: The Negative Exponent Rule
5.6: Power Rule For Exponents
5.7: The power of a product rule for exponents
5.8: Power of a quotient rule for exponents
5.9: Rational Exponents

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5.1: Definition of a^n

a^n Definition:

For any real number a and a positive number n, a^n is the repeated multiplication of a by itself n times.

$$a^n = a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \cdot a \ldots \ldots \cdot a$$

Notation:

a is the base, n is the positive exponent.

a^n is read as "a raised to the power of n."

Example 5.1.1

Identifying the base and exponent in expressions.

2^4, x^5, $\left(\frac{3}{7}\right)^7$, $(-3)^3$

Solution

Expression	Base	Exponent
2^4	2	4
x^5	x	5
$\left(\frac{3}{7}\right)^7$	$\frac{3}{7}$	7
$(-3)^3$	-3	3

Practice Problem

Identify the base and exponent of the following.

Expression	Base	Exponent
7^9		
$(-11)^6$		
a^b		
$\left(\frac{11}{12}\right)^5$		
12^3		
$\left(-\frac{7}{3}\right)^2$		
x^7		
$(2.56)^4$		

Evaluating expressions of the form a^n

When the base and exponent is a numerical value it is possible to evaluate an expression written in with an exponent. To find the value, use the definition and expand the expression. Once expanded, multiply and the result is the numerical value of the expression.

Example 5.1.2

Expand the following expressions and evaluate if possible.

3^4, $\left(\frac{3}{5}\right)^3$, x^7, $(3.12)^2$, $(-5)^3$, $(-y)^6$

Solution

3^4	$= 3 \cdot 3 \cdot 3 \cdot 3 = 81$
$\left(\frac{3}{5}\right)^3$	$\frac{3}{5} \cdot \frac{3}{5} \cdot \frac{3}{5} = \frac{27}{125}$
x^7	$x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x$ **Note: Can't evaluate since x is unknown**
$(3.12)^2$	$(3.12) \cdot (3.12) = 9.734$
$(-5)^3$	$-5 \cdot -5 \cdot -5 = -12$
$(-y)^6$	$-y \cdot -y \cdot -y \cdot -y \cdot -y \cdot -y = y^6$ **Note: y is unknown**

Exercise 5.1.1

Expand the following expressions and evaluate if possible.

1. 7^3
2. $\left(-\frac{2}{3}\right)^4$
3. $(-x)^7$
4. $(7.14)^2$
5. $(-3)^9$
6. $(z)^5$
7. $\left(-\frac{11}{33}\right)^2$
8. 6^5
9. $\left(\frac{x}{y}\right)^4$
10. a^{10}
11. $\left(\frac{2}{x}\right)^3$

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5.2: The Product Rule for Exponents

Definition: The Product Rule for Exponents

For any real number a and positive numbers m and n, the product rule for exponents is the following.

$a^m \cdot a^n = a^{m+n}$

Note: Bases must be the same to use the product rule.

5.2.1: Idea:

From the last section, $x^3 = x \cdot x \cdot x$ $\quad$ $x^5 = x \cdot x \cdot x \cdot x \cdot x$

Their product

$x^3 \cdot x^5 = x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x = x^8$

Hence, $x^3 \cdot x^5 = x^{3+5} = x^8$

Example 5.2.1

Use the product rule of exponents to simplify expressions.

1. $k^3 \cdot k^9$
2. $\left(\frac{2}{7}\right)^2 \cdot \left(\frac{2}{7}\right)^6$
3. $(-2a)^3 \cdot (-2a)^7$
4. $x \cdot x^3 \cdot x^{11}$
5. $y^{13} \cdot y^{33}$
6. $x^3 \cdot y^2 \cdot x \cdot y^4$

Solution

Expression	Product Rule	Base
$k^3 \cdot k^9$	$k^{3+9} = k^{12}$	k
$\left(\frac{2}{7}\right)^2 \cdot \left(\frac{2}{7}\right)^6$	$\left(\frac{2}{7}\right)^{2+6} = \left(\frac{2}{7}\right)^8$	$\frac{2}{7}$
$(-2a)^3 \cdot (-2a)^7$	$(-2a)^{3+7} = (-2a)^{10}$	$-2a$
$x \cdot x^3 \cdot x^{11}$	$x^{1+3+11} = x^{15}$	x
$y^{13} \cdot y^{33}$	$y^{13+33} = y^46$	y
$x^3 \cdot y^2 \cdot x \cdot y^4$	$x^{3+1} \cdot y^{2+4} = x^4 \cdot y^6$	x and y

Note: Again, the bases MUST be the same to simplify using the product rule of exponent

Helpful steps to simplify using the product rule of exponents:

1. Identify terms with common bases
2. Identify the exponent of common bases.
3. Add exponents of common bases and make the result of the sum the new exponent.
4. Repeat steps as need

Exercise 5.2.1

Use the product rule of exponents to simplify the following.

1. $f^3 \cdot f^11$

2. $\left(\frac{x}{7}\right)^2 \cdot \left(\frac{x}{7}\right)^3$
3. $(-7x)^9 \cdot (-7x)^7$
4. $h^5 \cdot h^3 \cdot h^{11}$
5. $t^{13} \cdot t^{33}$
6. $x^8 \cdot y^2 \cdot z \cdot x^3 \cdot y^2 \cdot z^{17}$
7. $x^3 \cdot y^4 \cdot x^3$

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5.3: The Quotient Rule of Exponents

Definition: The Quotient Rule for Exponents

For any real number a and positive numbers m and n, where $m > n$.

The Quotient Rule For Exponents is the following.

$$\frac{a^m}{a^n} = a^{m-n}$$

Note: Bases **MUST** be the same. Result will have the same base.

5.3.1: Idea:

From the last section,

$x^3 = x \cdot x \cdot x$ $\quad$ $x^5 = x \cdot x \cdot x \cdot x \cdot x$

Their quotient

$$\frac{x^5}{x^3} = \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x} = \frac{\cancel{x \cdot x \cdot x} \cdot x}{\cancel{x \cdot x \cdot x}} = \frac{x \cdot x}{1} = x \cdot x.$$

So, $\frac{x^5}{x^3} = x^{5-3} = x^2$

Example 5.3.1

Using the quotient rule of exponents to simplify expressions.

1. $\frac{k^3}{k^2}$
2. $\frac{r^{32}}{r^{21}}$
3. $\frac{\sqrt{2}^7}{\sqrt{2}^4}$
4. $\frac{(-7)^9}{(-7)^6}$
5. $\frac{(x\sqrt{5})^8}{x\sqrt{5}}$
6. $\frac{(xy)^{18}}{(xy)^{17}}$

Solution

Expression	Quotient Rule	Base
$\frac{k^3}{k^2}$	$k^{3-2} = k$	k
$\frac{r^{32}}{r^{21}}$	$r^{32-21} = r^{11}$	r
$\frac{\sqrt{2}^7}{\sqrt{2}^4}$	$\sqrt{2}^{7-4} = \sqrt{2}^3$	$\sqrt{2}$
$\frac{(-7)^9}{(-7)^6}$	$(-7)^{9-6} = (-7)^3$	-7
$\frac{(x\sqrt{5})^8}{x\sqrt{5}}$	$(x\sqrt{5})^{8-1} = (x\sqrt{5})^7$	$x\sqrt{5}$
$\frac{(xy)^{18}}{(xy)^{17}}$	$(xy)^{18-17} = xy$	xy

Note: In this section the exponent of the numerator was greater than the exponent of the denominator. That won't always be the case. The case where the exponent in the denominator is greater than the exponent in the numerator will be discussed in a later section.

? Exercise 5.3.1

Use the quotient rule of exponents to simplify the given expression.

1. $\frac{-y^{13}}{-y^{7}}$
2. $\frac{(2x)^{25}}{2x}$
3. $\frac{\sqrt{7}^{17}}{\sqrt{7}^{12}}$
4. $\frac{(-7)^{9}}{(-7)^{6}}$
5. $\frac{(x+y)^{78}}{(x+y)^{43}}$
6. $\frac{\sqrt{xy}^{15}}{\sqrt{xy}^{11}}$

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5.4: Zero Exponent Rule

In section 5.3, the exponent of the number in the numerator was always greater than the exponent of the number in the denominator.

In section 5.4, the exponent of the number in the numerator will be equal to the exponent of the number in the denominator.

Definition: The Zero Exponent Rule

For any real number a, The Zero Exponent Rule is the following

$a^0 = 1$

5.4.1: Idea:

From previous sections:

$$x^5 = x \cdot x \cdot x \cdot x \cdot x$$

and

$$\frac{x^5}{x^5} = \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x \cdot x \cdot x} = \frac{\cancel{x \cdot x \cdot x \cdot x \cdot x}}{\cancel{x \cdot x \cdot x \cdot x \cdot x}} = 1$$

Hence,

$$\frac{x^5}{x^5} = x^{5-5} = x^0 = 1$$

Example 5.4.1

Use the zero exponent rule to simplify expressions.

1. $\frac{x^9}{x^9}$
2. $\frac{d^5}{d^2 \cdot d^3}$
3. $\frac{5(xy)^3}{(xy)^3}$
4. $-\frac{y^3}{\sqrt{5}y^3}$
5. $\frac{(ab^2)^7}{(ab^2)^2 \cdot (ab^2)^4 \cdot (ab^2)}$

Solution

Expression	Zero Exponent Rule
$\frac{x^9}{x^9}$	$x^{9-9} = x^0 = 1$
$\frac{d^5}{d^2 \cdot d^3}$	$\frac{d^5}{d^{2+3}} = \frac{d^5}{d^5} = d^{5-5} = d^0 = 1$
$\frac{5(xy)^3}{(xy)^3}$	$5 \cdot \frac{(xy)^3}{(xy)^3} = 5 \cdot (xy)^{3-3} = 5 \cdot (xy)^0 = 5 \cdot 1 = 5$ The constant 5, can be factored out to see common bases clearly.
$-\frac{y^3}{\sqrt{5}y^3}$	$-\frac{1}{\sqrt{5}} \cdot \frac{y^3}{y^3} = -\frac{1}{\sqrt{5}} \cdot y^{3-3} = -\frac{1}{\sqrt{5}} \cdot y^0 = -\frac{1}{\sqrt{5}} \cdot 1 = -\frac{1}{\sqrt{5}}$ The constant $-\left(\frac{1}{\sqrt{5}}\right)$, can be factored out to see common bases clearly.

Expression	Zero Exponent Rule
$\frac{(ab^2)^7}{(ab^2)^2 \cdot (ab^2)^4 \cdot (ab^2)}$	$\frac{(ab^2)^7}{(ab^2)^{2+4+1}} = \frac{(ab^2)^7}{(ab^2)^7} = (ab^2)^{7-7} = (ab^2)^0 = 1$ First, simplify the denominator using the product rule of exponents. Then use the quotient rule of exponents to simplify the remaining expression.

Note: 0^0 does not equal 1. This is a special case that is covered in advanced courses. For now consider 0^0 to be undefined.

Helpful steps to simplify expressions with exponents

1. Identify common bases.
2. If needed combine common bases using the product rule of exponents.
3. If the expression contains common bases in both the numerator and denominator, use the quotient rule of exponents as needed.

Exercise 5.4.1

Use all the rules of exponents covered so far in this chapter to simplify the following.

1. $\frac{z^4}{z^4}$
2. $\frac{d^2 \cdot d^8}{d^7 \cdot d^3}$
3. $\frac{5(x+y)^3}{2(x+y)^3}$
4. $-\frac{\sqrt{9}y^3}{y^3}$
5. $\frac{(a^3b^2)^9}{(a^3b^2)^3 \cdot (a^3b^2)^4 \cdot (a^3b^2)^2}$
6. $\frac{(xyz)^{19}}{(xyz)^{19}}$

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5.5: The Negative Exponent Rule

In section 5.3, the exponent of the number in the numerator was greater than the exponent of the number in the denominator. In section 5.4, the exponent of the number in the numerator was equal to the exponent of the number in the denominator. In section 5.5, the exponent of the number in the denominator may be greater than the exponent of the number in the numerator.

Definition: The Negative Exponent Rule

For any non zero real number a and any integer n, the negative exponent rule is the following

$a^{-n} = \frac{1}{a^n} or \frac{1}{a^{-n}} = a^n$

It is poor form in mathematics to leave negative exponents in the answer. All answers will always be simplified to show positive exponents.

How does this work?

Recall:

$$2^3 = 2 \cdot 2 \cdot 2 = 8$$
$$2^2 = 2 \cdot 2 = 4$$
$$2^1 = 2 = 2$$
$$2^0 = 1$$

What happens with negative exponents?

$$2^{-1} = \frac{1}{2^1} = \frac{1}{2}$$
$$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$$

Recall: From the last section,

$$x^3 = x \cdot x \cdot x$$
$$x^5 = x \cdot x \cdot x \cdot x \cdot x$$

Their quotient:

$$\frac{x^3}{x^5} = \frac{x \cdot x \cdot x}{x \cdot x \cdot x \cdot x \cdot x} = \frac{\cancel{x \cdot x \cdot x}}{\cancel{x \cdot x \cdot x} \cdot x \cdot x} = \frac{1}{x \cdot x} = \frac{1}{x^2}$$

Apply the quotient rule to obtain an equivalent result.

$\frac{x^3}{x^5} = x^{3-5} = x^{-2}$

Using the negative exponent rule.

$x^{-2} = \frac{1}{x^2}$.

Review the following examples to help understand the process of simplifying using the quotient rule of exponents and the negative exponent rule.

Hint: Be patient, take your time and be careful when simplifying!

Example 5.5.1

Simplify the following expression to a single base with only positive exponents.

$\frac{t^5}{t^1 1}$

Solution

$t^{5-11} = t^{-6} = \frac{1}{t^6}$

Example 5.5.2

Simplify the following expression to a single base with only positive exponents.

$\frac{x^3 \cdot x^{11}}{x \cdot x^{15}}$

Solution

$\frac{x^{3+11}}{x^{1+15}} = \frac{x^14}{x^16} = x^{14-16} = x^{-2} = \frac{1}{x^2}$

Example 5.5.3

Simplify the following expression to a single base with only positive exponents.

$\frac{2y^3}{7y^7}$

Solution

$\frac{2}{7} \cdot \frac{y^3}{y^7} = \frac{2}{7} \cdot y^{3-7} = \frac{2}{7} \cdot y^{-4} = \frac{2}{7} \cdot \frac{1}{y^4} = \frac{2}{7y^4}$

Example 5.5.4

Simplify the following expression to a single base with only positive exponents.

$-\frac{\sqrt{3}z^6}{z^7}$

Solution

$-\sqrt{3} \cdot \frac{z^6}{z^7} = -\sqrt{3} \cdot z^{6-7} = -\sqrt{3} \cdot z^{-1} = -\sqrt{3} \cdot \frac{1}{z} = -\frac{\sqrt{3}}{z}$

In Examples 3 and 4, factor out the constant to see the common bases clearly.

Example 5.5.5

Simplify the following expression to a single base with only positive exponents.

$\frac{1}{a^{-9}}$

Solution

a^9

Example 5.5.6

Simplify the following expression to a single base with only positive exponents.

$\frac{x^3}{x^{-5}}$

Solution

$x^{3-(-5)} = x^{3+5} = x^8$

Example 5.5.7

Simplify the following expression to a single base with only positive exponents.

$$\frac{c^{-7}}{c^{-3}}$$

Solution

$c^{(-7)-(-3)} = c^{-7+3} = c^{-4} = \frac{1}{c^4}$

In examples 6 and 7, the quotient rule of exponents was used before changing exponents to positive exponents. The same results are obtained by expanding and changing exponents to positive exponents first and then applying the quotient rule of exponents.

Exercise 5.5.1

Simplify the following expressions to a single base with only positive exponents.

1. $\frac{p^4}{p^{13}}$
2. $-\frac{k^2 \cdot k^3}{k^7 \cdot k^8}$
3. $\frac{5(x+y)^3}{2(x+y)^{13}}$
4. $-\frac{\sqrt{8}y^3}{y^{-3}}$
5. $\frac{a^{-7}}{a^2 \div a^{-5}}$
6. $\frac{x^{-7}}{x^5}$

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5.6: Power Rule For Exponents

This rule helps to simplify an exponential expression raised to a power. This rule is often confused with the product rule, so understanding this rule is important to successfully simplify exponential expressions.

Definition: The Power Rule For Exponents

For any real number a and any numbers m and n, the power rule for exponents is the following:

$(a^m)^n = a^{m \cdot n}$

Idea:

Given the expression

$(2^2)^3$ Use the exponent definition to expand the expression inside the parentheses.

$(2 \cdot 2)^3$ Now use the exponent definition to expand according to the exponent outside the parenth

$(2 \cdot 2) \cdot (2 \cdot 2) \cdot (2 \cdot 2) = 2^6$ $= 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^{1+1+1+1+1+1} = 2^6$ (Product Rule of Exponents)

Hence, $(2^2)^3 = 2^{2 \cdot 3} = 2^6$

Example 5.6.1

Simplify the following expression using the power rule for exponents.

$(-3^4)^3$

Solution

$(-3)^{4 \cdot 3} = (-3)^{12}$

Example 5.6.2

Simplify the following expression using the power rule for exponents.

$(-3^4)^3$

Solution

$(5y)^{3 \cdot 7} = (5y)^{21}$

Example 5.6.3

Simplify the following expression using the power rule for exponents.

$((-y)^5)^2$

Solution

$(-y)^{5 \cdot 2} = (-y)^{10} = y^{10}$

Example 5.6.4

Simplify the following expression using the power rule for exponents.

$(x^{-2})^3$

Solution

$x^{-2 \cdot 3} = x^{-6} = \frac{1}{x^6}$

Hint: Parentheses in the problem is a strong indicator of simplifying using the power rule for exponents.

Exercise 5.6.1

Simplify the expression using the power rule for exponents.

1. $(x^3)^5$
2. $((-y)^3)^7$
3. $((-6y)^8)^{-3}$
4. $(x^{-2})^{-3}$
5. $(r^4)^5$
6. $(-p^7)^7$
7. $((3k)^{-3})^5$

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5.7: The power of a product rule for exponents

The power of a product rule for exponents will deal with expressions where a product of bases is raised to some power.

Definition: The Power of a Product Rule for Exponents

For any real number a and b and any number n, the power of a product rule for exponents is the following:

$(a \cdot b)^n = a^n \cdot b^n$

Example 5.7.1

Simplify the following expression

$(a \cdot b)^3$

Solution

$(a \cdot b)3$	Given
$a \cdot b \cdot a \cdot b \cdot a \cdot b$	Expand using exponent 3
$a \cdot a \cdot a \cdot b \cdot b \cdot b$	Reorder product using the commutative property.
a^3b^3	Simplify to single base.

Example 5.7.2

Simplifying expression using the power of a product rule for exponents.

$(2x^3y^2)^2$

Solution

$((2x^3y^2)^2)$	Given
$= 2^2 \cdot x^{3 \cdot 2} \cdot y^{2 \cdot 2}$	Power of a product rule applied
$= 4x^6y^4$	Simplify by multiplying exponents

Example 5.7.3

Simplifying expression using the power of a product rule for exponents.

$(2x^5 \cdot 3y)^3$

Solution

$(2x^5 \cdot 3y)^3$	Given
$= (6x^5y)^3$	If possible, simplify inside the parentheses.
$= 6^3 \cdot x^{5 \cdot 3} \cdot y^3$	Power of a product rule for exponents applied.
$216x^{15}y^3$	Simplify by multiplying as needed.

Example 5.7.4

Simplifying expression using the power of a product rule for exponents.

$(3ab^4)^{-2}$

Solution

$(3ab^4)^{-2}$	Given
$= \frac{1}{(3ab^4)^2}$	Negative exponent rule applied
$= \frac{1}{3^2 \cdot a^3 \cdot b^{4 \cdot 2}}$	Power of a product rule applied.
$\frac{1}{9a^3b^8}$	Simplify by multiplying as needed.

Exercise 5.7.1

Simplify the expression using the power of a product rule for exponents.

1. $(xy^2)^3$
2. $(2xy^2z^3)^7$
3. $(x^2)^{-3}$
4. $(x^2y)^{-3}$
5. $(2r^4)^5$
6. $(-2p^7)^7$
7. $(3k \cdot 2j^2)^5$

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5.8: Power of a quotient rule for exponents

The power of a quotient rule for exponents will focus on what happens to a quotient when it is raised to some power.

Definition: The Power of a Quotient Rule for Exponents

For any real number a and b and any integer n, the power of a quotient rule for exponents is the following:

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$,

where $b \neq 0$.

Simplify the following using power of a quotient rule for exponents.

Example 5.8.1

Simplify the following using power of a quotient rule for exponents.

$\left(\frac{a}{b}\right)^4$

Solution

$\left(\frac{a}{b}\right)^4$	Given
$= \frac{a}{b} \cdot \frac{a}{b} \cdot \frac{a}{b} \cdot \frac{a}{b}$	Expand using the exponent definition
$= \frac{a^4}{b^4}$	Multiply as needed to simplify

Example 5.8.2

$\left(\frac{x^2}{3y^5}\right)^3$

Solution

$\left(\frac{x^2}{3y^5}\right)^3$	Given
$= \frac{x^{2 \cdot 3}}{3^3 \cdot y^{5 \cdot 3}}$	power of quotient rule for exponents applied
$= \frac{x^6}{3^3 \cdot y^{15}}$	Simplify exponent product
$= \frac{x^6}{27y^{15}}$	Multiply as needed to simplify.

Example 5.8.3

$\left(\frac{2x}{y}\right)^{-3}$

Solution

$\left(\frac{2x}{y}\right)^{-3}$	Given
$= \left(\frac{y}{2x}\right)^3$	Negative exponent rule applied
$= \frac{y^3}{2^3 \cdot x^3}$	Power of a quotient rule for exponents applied.
$= \frac{y^3}{8x^3}$	Multiply as needed to simplify.

The order in which rules of exponents are applied does not matter. In example three, steps 2 and 3 can be done in any order. The results will be the same.

Exercise 5.8.1

Simplify the expression using the power of a quotient rule for exponents.

1. $\left(\frac{p^4}{p^7}\right)^3$
2. $-\left(\frac{x^2 \cdot x^3}{x \cdot y^3}\right)^2$
3. $\left(\frac{5x^3}{2y^{13}}\right)^{-2}$
4. $\left(\frac{2c^3}{c^4}\right)^3$
5. $\left(\frac{a^{-7}b}{a^2b^{-4}}\right)^3$
6. $\left(\frac{f^{-7}}{f^5}\right)^9$
7. $\left(\frac{xy^2z^3}{x^3y^2z}\right)^5$

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5.9: Rational Exponents

Exponents are not always integers. This section will look into the cases where an exponent is a rational number. When an exponent is a rational number, the expression may be written as an expression with a radical. The rule is to write your answer in the same form as the original problem (if you start with exponents, end with exponents, or if you start with radicals, end with radicals).

1/n Definition: Rational Exponents of the Form 1/n

For any real number a and any integer number n, an expression with the exponent of $\frac{1}{n}$ may be expressed as the following

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

Note: n is the index in the radical. $\sqrt[n]{a}$ is read "*the nth root of a*"

Note: When the radical does not have a visible index, by default the index is 2 (square root). Indices greater than 2 will be marked on the radical.

Example 5.9.1

1. $(4)^{\frac{1}{2}} = \sqrt{4} = 2$ Index is 2 by default
2. $(x)^{\frac{1}{7}} = \sqrt[7]{x}$ Index is 7
3. $(-3y)^{\frac{1}{3}} = \sqrt[3]{(-3y)}$ Index is 3

Now, Let's observe what happens when the exponent is a rational number with numerator $\neq 1$.

m/n Definition: Rational Exponents of the Form m/n

For any real number a and any integer number n and m, an expression with the exponent of $\frac{m}{n}$ may be expressed as the following

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \text{ or } (\sqrt[n]{a})^m$$

Note: n is the index in the radical and m is the power of the base.

Example 5.9.2

Write the following in radical form

1. $(x)^{\frac{2}{3}} = \sqrt[3]{x^2} = (\sqrt[3]{x})^2$ Index is 3 and base is raised to the power of 2.
2. $(5t)^{\frac{7}{8}} = \sqrt[8]{5t^7} = (\sqrt[8]{5t})^7$ Index is 8 and base is raised to the 7 power.
3. $(x)^{\frac{2}{3}} = \sqrt[3]{x^2} = (\sqrt[3]{x})^2$ Index is 3 and base raised to the power 2.
4. Illegal pream-token (&)
5. $\left(\frac{3}{4}\right)^{\frac{5}{7}} = \sqrt[7]{\frac{3}{4}^5}$ Rational exponent written as radical with index 7 and base raised to the power of 5.

? Exercise 5.9.1

Write the following in radical form.

1. $(x)^{\frac{5}{7}}$
2. $(xy)^{\frac{9}{8}}$
3. $(x)^{\frac{9}{5}}$
4. $(z)^{-\frac{11}{13}}$
5. $\left(\frac{x}{4}\right)^{\frac{6}{9}}$
6. $6(y)^{\frac{1}{17}}$

7. $(6y)^{\frac{1}{17}}$
8. $\left(\frac{3}{4}\right)^{\frac{x}{y}}$
9. $\left(\frac{7}{4}\right)^{\left(-\frac{x}{y}\right)}$

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CHAPTER OVERVIEW

6: Absolute Value

6.1: Evaluating Expressions

6.2: Solving Absolute Value Equations

6.3: Solving Absolute Value Inequalities and Writing Answers in Interval Notation

Thumbnail: The graph of the absolute value function for real numbers. (CC BY-SA 3.0; Qef and Ævar Arnfjörð Bjarmason via Wikipedia).

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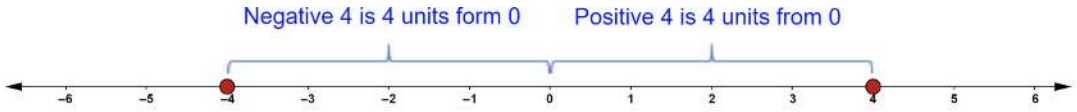
6.1: Evaluating Expressions

Definition: Absolute Value

The **absolute value** of a real number a, written $|a|$, is the distance from a to 0 on a number line.

To find $|-4|$, ask: "what is the distance from -4 to 0?". Draw a number line and see that $|-4|=4$. Similarly, $|4|=4$, as shown in the figure below.



Example 6.1.1

Evaluate the following expressions:

a. $|8-2|-|4-7|$
b. $5|-3|+|-9|^2$
c. $\frac{3}{5}|6+(-3)^3|$
d. $\left|\frac{(-2)^2+12}{3}+5\right|+|-4+2|$

Solution

a. To evaluate $|8-2|-|4-7|$, first simplify inside the absolute value.

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b. First, simplify the absolute values, then apply the required arithmetic operation.

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c. Use order of operations "PEMDAS" to simplify inside the absolute value.

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d. To evaluate the expression in this part, first apply the order of operation "PEMDAS" inside the absolute value to simplify.

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Exercise 6.1.1

Evaluate the given expressions:

1. $|8-15|$
2. $|-3-12|$
3. $\left|-2+11-\left(-\frac{6}{4}\right)\right|$
4. $\left|-\frac{1+5}{12}-5\right|-1$
5. $|2(5+6)-20|$
6. $\left|\frac{1}{2}(21-5)-|(-2)^3\right|$
7. $|-5|-2(-13+10)|$
8. $\frac{3}{2}\left|12\left(\frac{-7+17}{(6-2)}\right)\right|+|-(-2)|$

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6.2: Solving Absolute Value Equations

To solve absolute value equations, first consider the following two properties of **absolute value**:

Definition: Properties of Absolute Value

Property 1: For $b > 0$, $|a| = b$ if and only if $a = b$ or $a = -b$

Property 2: For any real numbers a and b, $|a| = |b|$ if and only if $a = b$ or $a = -b$

- Before Property 1 is applied, **isolate** the absolute value expression to either side of the equation.
- Check the solutions by substituting them back into the original equation.
- Solutions are presented as a solution set of the form $\{p, q\}$, where p and q are any real numbers.
- The solution set of an absolute value equation is graphed as points on a number line.

Example 6.2.1

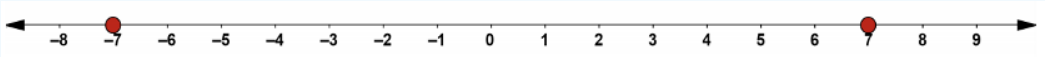
Solve each equation and graph the solution set.

a. $|x| = 7$
b. $|5x–3| = 2$
c. $|20–x| = -80$

Solution

a. To solve $|x| = 7$, apply Property 1 with $a = x$ and $b = 7$.

Therefore, the solutions are, $x = -7$ and $x = 7$, and the solution set is $\{-7, 7\}$. The graph of the solution set is as shown in the figure below.



b. The equation-solving method used in part a can be extended to the given equation in this part with $a = 5x–3$ and $b = 2$.

Thus, the absolute value equation $|5x–3| = 2$ is equivalent to:

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Now, check if $x = 1$ and $x = \frac{1}{5}$ are solutions to the given absolute value equation.

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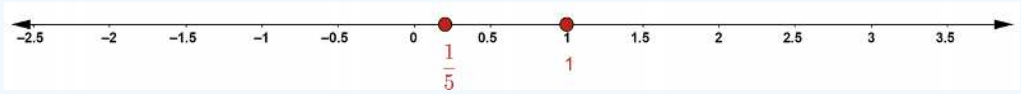
Since the above equations are true, then, $x = 1$ and $x = \frac{1}{5}$ are solutions to the given absolute value equation. The solution set is $\left\{\frac{1}{5}, 1\right\}$. The graph of the solution set is as shown in the figure below.



c. Since an absolute value can never be negative, there are no real numbers x that makes $|20–x| = -80$ true. The equation has no solution and the solution set is $\emptyset$.

Example 6.2.2

Solve and graph the solution set.

a. $\left|\frac{4}{3}x + 3\right| + 8 = 18$
b. $4\left|\frac{1}{3}x - 6\right| - 5 = -5$
c. $|4x–3| = |x + 6|$

Solution

a. Notice that the absolute value expression is not isolated which means the properties cannot be applied. First, isolate $\left|\frac{4}{3}x+3\right|$ on the left side of the equation, then, apply Property 1.

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With the absolute value now isolated, solve $\left|\frac{4}{3}+3\right|=10$ using Property 1, with $a=\frac{4}{3}x+3$ and $b=10$ as follows,

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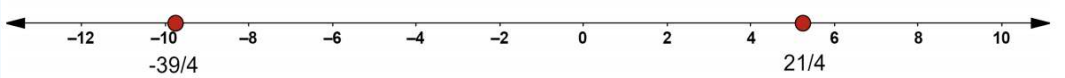
Check the solutions $x=-\frac{39}{4}$ and $x=\frac{21}{4}$ by substituting them into the original absolute value equation. The solution set is $\left\{-\frac{39}{4},\frac{21}{4}\right\}$ and the graph of the solution set is as shown in the figure below.



b. Similar to part a, isolate the absolute value expression. So, first isolate $\left|\frac{1}{3}x-6\right|$ on the left side of the equation and apply Property 1.

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The absolute value is isolated. Since 0 is the only number whose absolute value is 0, the expression $\frac{1}{3}x-6$ must be equal to 0. So,

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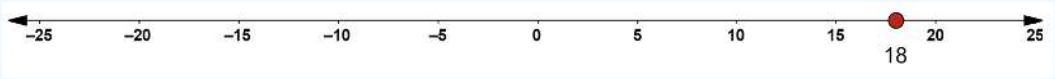
The solution is 18 and the solution set is $\{18\}$. Verify that it satisfies the original equation. The graph of the solution set is as shown in the figure below.



c. $|4x-7|=|x+14|$ Notice that to solve $|4x-7|=|x+14|$, use Property 2 with $a=4x-7$ and $b=x+14$.

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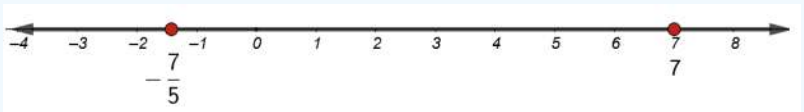
Check the solutions $x=-\frac{7}{5}$ and $x=7$ by substituting them into the original absolute value equation. The solution set is $\left\{-\frac{7}{5},7\right\}$. The graph of the solution is as shown in the figure below.



Exercise 6.2.1

Solve each equation, check the solution and graph the solution set.

1. $|x|=19$
2. $|x-4|=10$
3. $|2x-5|=12$
4. $\left|\frac{x}{11}\right|=2.5$
5. $|x-3.8|=-2.7$
6. $|3x-4.5|=9.3$
7. $\frac{8}{3}|x-6|=14$
8. $|x+15|-19=7$
9. $|11x+3|+28=16$

10. $\left|\frac{8}{7}x+9\right|-2=8$
11. $-3|2x-7|+13=13$
12. $8-5|10x+6|=5$
13. $|5x-14|=|3x-9|$
14. $|15x|=|x-21|$
15. $|4x-7|=|5(2x+3)|$
16. $\frac{7}{8}=\frac{3x}{2}+\frac{2x}{5}$

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6.3: Solving Absolute Value Inequalities and Writing Answers in Interval Notation

The previous section taught how to solve absolute value equations. This section teaches how to solve absolute value inequalities. To do so, first consider the following two properties:

Definition: Properties of Absolute Value Inequalities

Property 1: For all positive number b, and all real numbers p and q,

i. $|a| < b$ if and only if $-b < a < b$.

The solution set is of the form (p, q), a single open interval.

ii. $|a| \leq b$ if and only if $-b \leq a \leq b$.

The solution set is of the form $[p, q]$, a single closed interval.

Before considering Property 2, it is important to define the union of two intervals. The union of any two intervals A and B, is the set of elements in A, or B, or both. The union is represented with the symbol $\cup$.

Property 2: For all positive number b, and all real numbers p and q,

i. $|a| > b$ if and only if $a < -b$ or $a > -b$

The solution set is of the form $(-\infty, p) \cup (q, \infty)$, a disjoint interval.

ii. $|a| \geq b$ if and only if $a \leq -b$ or $a \geq b$.

The solution set is of the form $(-\infty, p] \cup [q, \infty)$, a disjoint interval.

Note that before the properties of inequalities are applied, **isolate** the absolute value expression on either side of the inequality.

Example 6.3.1

Solve the following inequalities and graph the solution set.

a. $|5x - 2| < 7$
b. $|8x - 6| < -1$
c. $2|x - 3| + 5 \leq 9$

Solution

a. This is an absolute value expression less than a positive number of the form $|a| < b$. Apply Property 1(i) with $a = 5x - 2$ and $b = 7$.

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To solve the inequality, isolate x. The previous step becomes,

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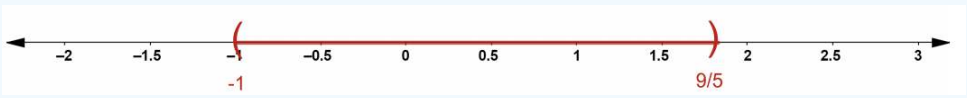
The Solution set is the single open interval $\left(-1, \frac{9}{5}\right)$ and the graph is as shown in the figure below.



b. Recall that the absolute value of any number is the distance from 0 to that number on the number line. This means, the absolute value of any number is always greater or equal to 0.

This example gives $|8x - 6| < -1$, which can not happen since a distance is never negative. So, the absolute value inequality has no solution and the solution set is the empty set, written ϕ.

c. To solve $2|x - 3| + 5 \leq 9$, isolate the absolute value.

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Now, $|x-3| \leq 2$ is of the form $|a| \leq b$. Apply Property 1 (ii) with $a = x-3$ and $b = 2$.

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The solution set is the single interval $[1, 5]$ and the graph is as shown in the figure below.

Example 6.3.2

Solve and graph the solution set.

a. $\left|\frac{6-x}{10}\right| \geq 3$
b. $2 < \left|\frac{3}{4}x - 3\right| - 5$
c. $|2-4x| \geq -7$

Solution

a. The absolute value inequality $\left|\frac{6-x}{10}\right| \geq 3$ is in the form of $|a| \geq b$. Apply Property 2 (ii) with $a = \frac{6-x}{10}$ and $b = 3$ to solve the inequality.

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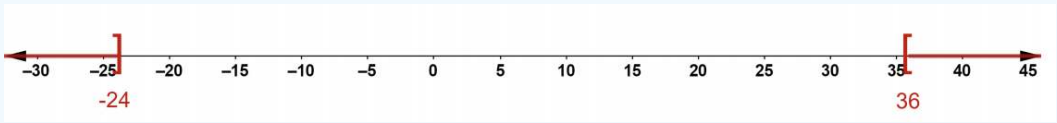
Note that since the inequalities were multiplied by a negative number, namely -1, the direction of the inequality changed.

The solution set is the union of the two intervals. Thus, $(-\infty, -24] \cup [36, \infty)$ is the solution set in interval notation. The graph of the solution is as shown in the figure below.



b. Isolate the absolute value.

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Note that the above inequality is read from right to left as "the absolute value of the expression $\frac{3}{4}x - 3$ is greater than 7" or equivalently switch the order of the absolute value inequality to have $\frac{3}{4}x - 3 > 7$, which is a more familiar form to solve.

Now, $\frac{3}{4}x - 3 > 7$ is of the form $|a| > b$. Use Property 2 (ii) with $a = \frac{3x}{4} - 3$ and $b = 7$.

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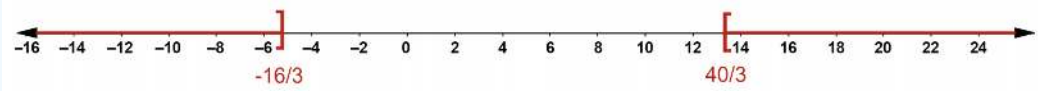
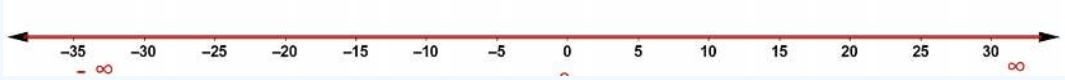
The solution set is the union of the two intervals, $(-\infty, -\frac{16}{3}] \cup [\frac{40}{3}, \infty)$. The graph of the solution is as shown in the figure below



c. Since $|2-4x|$ is always greater than or equal to 0 for any real numbers x then, the absolute value inequality is true for all real numbers. Let x be any real number, negative or positive, then the absolute value will either be 0 or a positive number.

So, the solution set is all the real numbers on the number line, as shown in the figure below. The solution set in interval notation is $(-\infty, \infty)$.



Exercise 6.3.1

Solve the following inequalities, write answers in interval notation, and graph the solution sets:

1. $|-6x+1|<20$
2. $\left|\frac{2}{3}x+5\right|>5$
3. $\left|5-\frac{1}{4}x\right|<-71$
4. $2\left|-x+\frac{4}{5}\right|\leq\frac{5}{2}$
5. $-\frac{1}{7}<|x+10|-10$
6. $|-12-3x|<-0.6$
7. $\left|\frac{16-2x}{8}\right|\geq 11$
8. $|2-6x|-5\geq -9$
9. $\left|\frac{2}{3}x-\frac{1}{4}\right|\leq\frac{1}{12}$
10. $|.02x+5|<.02$
11. $\left|\frac{1}{2}-x\right|<8$
12. $|-6x+9|-5<-6$

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CHAPTER OVERVIEW

7: Straight Lines

7.1: Slope of a Line
7.2: Parallel Lines
7.3: Perpendicular Lines
7.4: Equations of Vertical and Horizontal Lines
7.5: Forms of the Equation of a Line
7.6: Applied Examples

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7.1: Slope of a Line

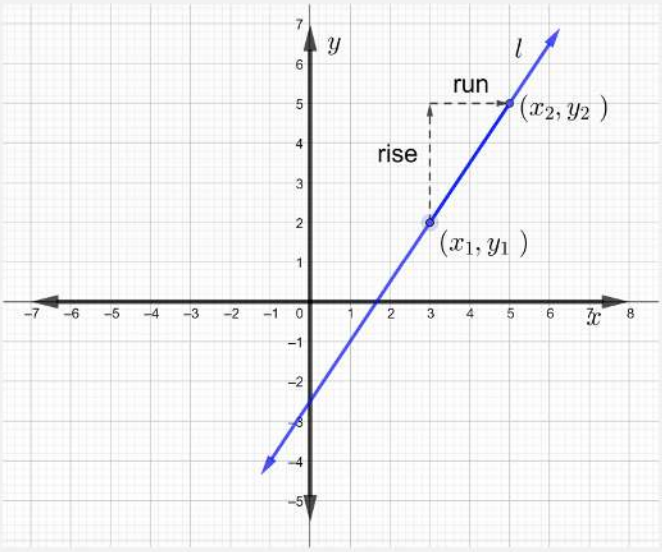
Recall that ordered pairs can be graphed as points in the rectangular coordinate plane. Through any two distinct points, graph a straight line, l.

To describe line l algebraically, first consider the slope of a line formula.

Definition: Slope of a Line

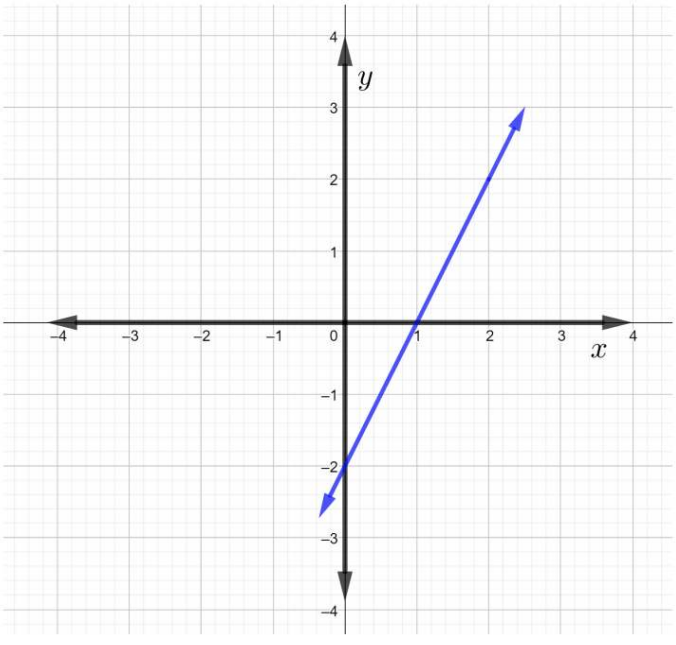
The slope m of a line l that passes through points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} \text{ where } x_2 \neq x_1$$



Example 7.1.1

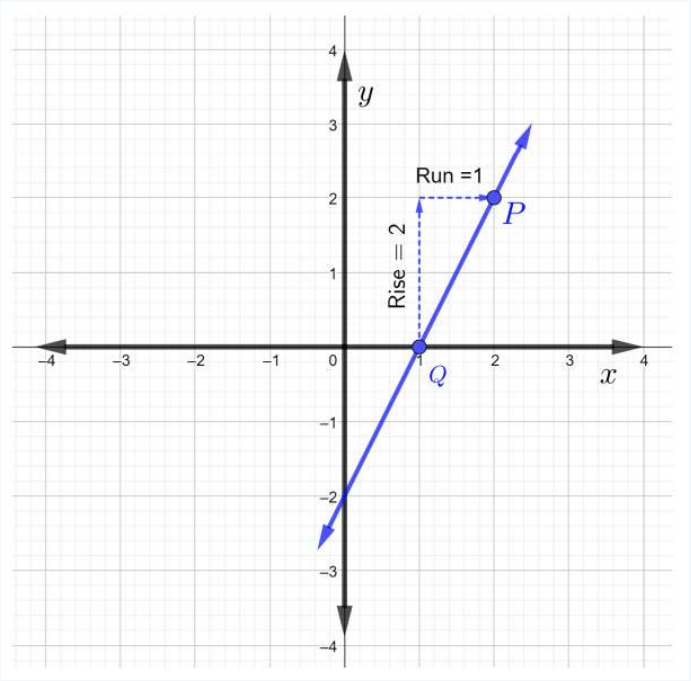
Find the slope of the line in the figure below.



Solution

By the above definition of the slope of the line formula, the slope of the line can be written as $m = \frac{\text{rise}}{\text{run}}$. Start by choosing any two points, P and Q, on the line. Choose point P to be $(2, 2)$ and point Q to be $(1, 0)$.

Starting at point Q, rise up to point P by counting upward 2 grid squares, which means $\text{rise} = 2$. Now, to arrive to point P, run 1 grid square to the right, which means that $\text{run} = 1$, as shown in the figure below.



Thus,

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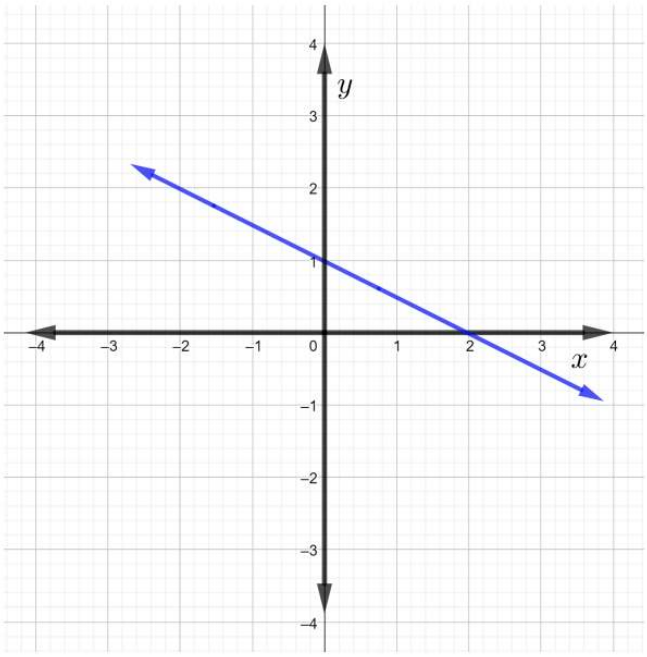
Therefore, the slope of the line in the figure is $m = 2$.

Example 7.1.2

Find the slope of the line shown in the figure below.

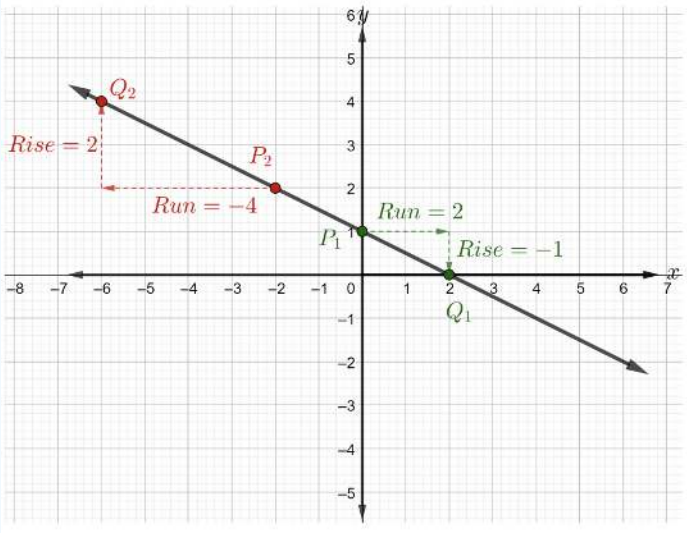


Solution

Similar to Example 1, start by choosing any two points, P and Q, on the line.

Note: Since any 2 points on the line can be chosen, it will be easier to choose the two points that are integers. Those points are located on the line and also on the intersection of two grid lines. For example, in the figure it will be easier to choose any two of the following points on the given line: $(2,0)$, $(0,1)$, $(4,-1)$, $(6,-2)$, $(-4,3)$, $(-6,4)$, and so on ...

The slope is the same for any two points P and Q on the line. Choose point P_1 to be $(0,1)$ and point Q_1 to be $(2,0)$. Starting at point P_1, reach point Q_1 by first running 2 grid squares to the right, which means that the $\text{run} = 2$. Now, to arrive to point Q_1 count downward 1 grid square. Notice, that the $\text{rise} = -1$ which means to move down 1 unit as shown in the figure below.



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Therefore, the slope of the line in the figure above is $m = -\frac{1}{2}$.

Now, choose point P_2 to be $(-2,2)$ and point Q_2 to be $(-6,4)$ as shown in the above figure. Starting at points P_2, reach point Q_2 by first running 4 grid squares to the left, which means that the $\text{run} = -4$. Now, to arrive to point Q_2 count upward 2 grid

squares. Thus, the $\text{rise} = 2$. The slope is $m = \frac{2}{-4} = -\frac{1}{2}$. Notice, the slope is the same regardless of which 2 points we consider on a given line.

Example 7.1.3

Find the slope of the line that passes through $(3, 2)$ and $(4, 4)$ using the slope formula. Graph the line that passes through the given points.

Note: The order the points are labeled will make no difference in the slope of a line formula as long as there is consistency.

Solution

Let $(x_1, y_1) = (3, 2)$ and $(x_2, y_2) = (4, 4)$ then,

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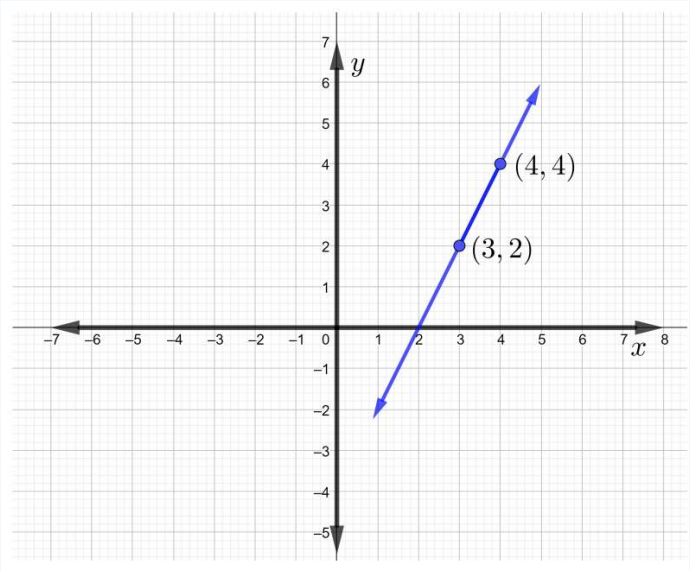
Therefore, the slope of the line passing through points $(3, 2)$ and $(4, 4)$ is $m = 2$. The line that passes through the given points is as shown in the figure below.



Notice that when the line rises from left to right, the line has a **positive** slope.

Example 7.1.4

Find the slope of the line that passes through the points $(-1, 2)$ and $(3, -4)$. Plot the points and graph the line.

Solution

Let $(x_1, y_1) = (-1, 2)$ and $(x_2, y_2) = (3, -4)$ then,

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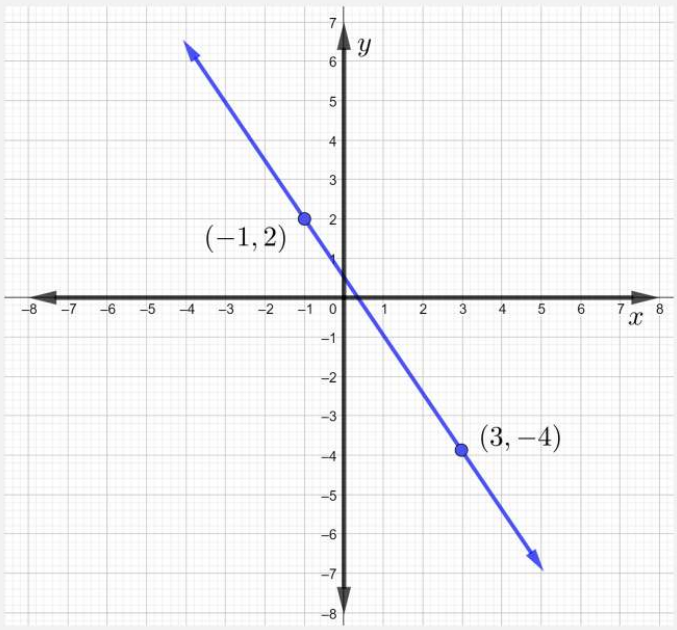
Definition: Negative Slope

Now, to graph the line through the given points, first plot the two points, then draw a straight line through them, as shown in the Figure below.

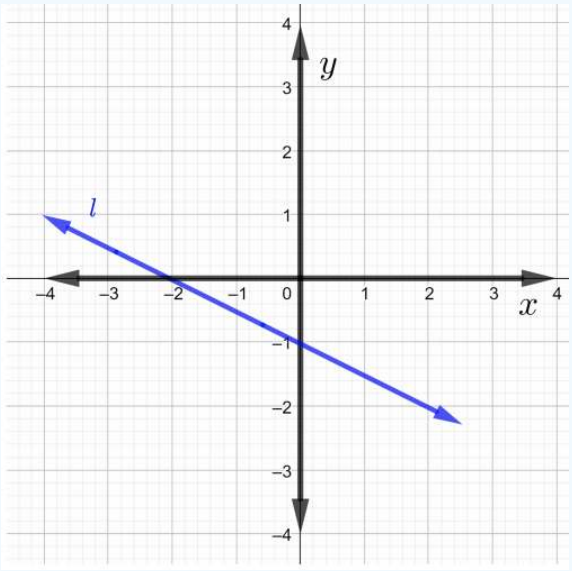


Notice that when the line falls from left to right, the line has a **negative** slope.

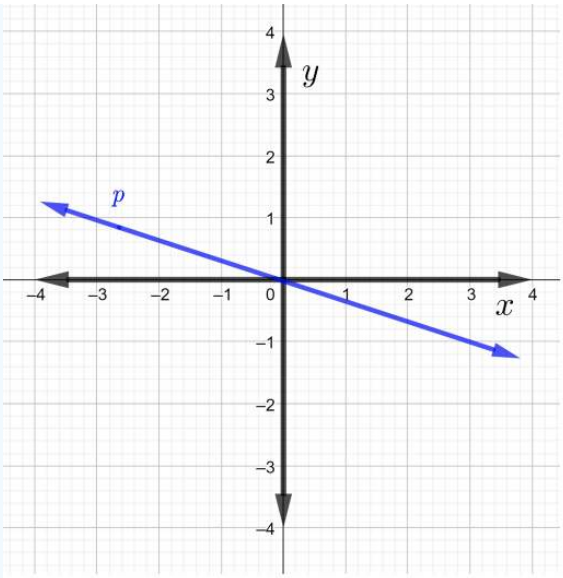
Exercise 7.1.1

Find the slopes of each line in the graph in Problems 1 through 4

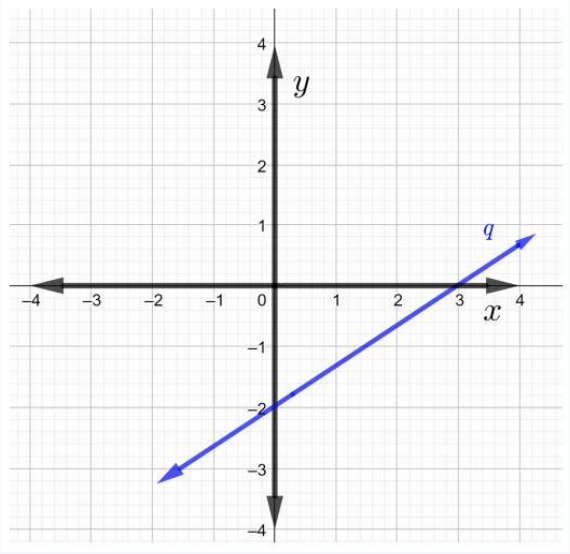
1.



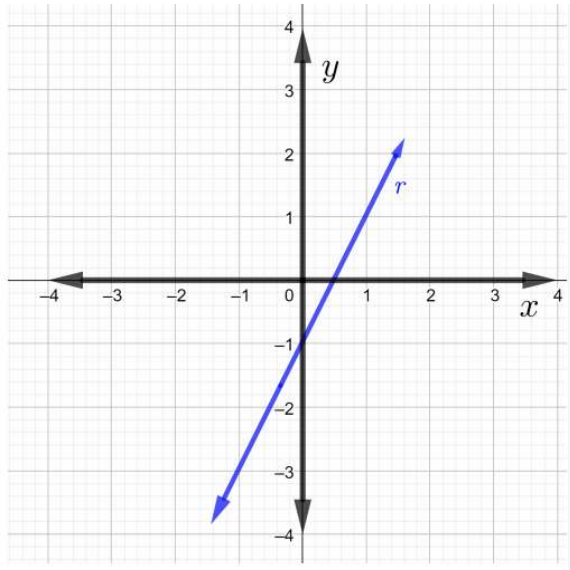
2.



3.



4.



Exercise 7.1.2

Find the slope of the line that passes through the given points in Problems 5 through 7.

5. $(-3, 5)$ and $(4, -5)$
6. $(2, 5)$ and $(0, -1)$
7. $(4, 1)$ and $(0, 0)$

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7.2: Parallel Lines

Definition: Parallel Lines

In a coordinate plane, **parallel lines** are lines that do not meet or intersect. They are always the same distance apart. Moreover, parallel lines have the *same slope*.

Example 7.2.1

Find the slope of the line l that passes through $(2,0)$ and $(4,-3)$ and the slope of the line q that passes through $(2,-3)$ and $(4,-6)$. Determine if the lines are parallel.

Solution

Use the slope of the line formula to find the slope of line l, m_l, and the slope of line q, m_q, as follows,

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Since the two slopes are equal, then, lines l and q are parallel.

Exercise 7.2.1

Determine whether the given lines are parallel:

a. The line l that passes through the points $(2,2)$ and $(3,3)$ and the line q that passes through the points $(4,1)$ and $(0,5)$.
b. The line l that passes through the points $(1,3)$ and $(6,-2)$ and the line q that passes through the points $(-2,-7)$ and $(10,5)$.
c. The line l that passes through the points $(-6,5)$ and $(2,-1)$ and the line q that passes through the points $(-4,0)$ and $(0,-3)$.

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7.3: Perpendicular Lines

Definition: Perpendicular Lines

Two distinct lines l and q are **perpendicular**, written $l \perp q$, if their intersection form four right angles or angles with measure 90°. The slopes of the perpendicular lines l and q are negative reciprocals. That is,

$$m_l = -\frac{1}{m_q}$$

and

$$m_q = -\frac{1}{m_l}$$

Example 7.3.1

Determine if the given lines are perpendicular. The line l that passes through the points $(0,1)$ and $(1,3)$, and the line q that passes through the points $(-1,4)$ and $(5,1)$.

Solution

To determine if the lines are perpendicular, first find their slopes using the slope of the line formula. The slope of line l, m_l, that passes through the points $(0,1)$ and $(1,3)$ is,

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The slope of line q, m_q, that passes through the points $(-1,4)$ and $(5,1)$, is

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Now, lines l and q are perpendicular if and only if:

$$m_l = -\frac{1}{m_q} \text{ and } m_q = -\frac{1}{m_l}$$

$m_l = 2$ and $m_q = -\frac{1}{m_l} = -\frac{1}{2}$. Hence, the slopes of the lines are negative reciprocals so it can be concluded that lines l and q are perpendicular lines.

Example 7.3.2

Find the slope of a line perpendicular to line l that passes through the points $(-3,0)$ and $(3,4)$.

Solution

Start by finding the slope of line l that passes through the points $(-3,0)$ and $(3,4)$, using the slope of the line formula. Thus,

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Any line perpendicular to line l must have a slope that is negative reciprocal of its slope. Since $m_l = \frac{2}{3}$ then the slope of the line perpendicular to line l must be $m = -\frac{3}{2}$

Exercise 7.3.1

Determine if the given lines are perpendicular.

a. The line l that passes through the points $(0,4)$ and $(5,3)$ and the line q that passes through the points $(1,5)$ and $(-1,-5)$.
b. The line l that passes through the points $(-2,-5)$ and $(1,7)$ and the line q that passes through the points $(-4,1)$ and $(-3,-3)$.

Exercise 7.3.2

Find the slope of a line perpendicular to:

a. Line l that passes through the points $(4, 2)$ and $(-1, -2)$.
b. Line q that passes through the points $(7, -8)$ and $(9, 1)$.

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7.4: Equations of Vertical and Horizontal Lines

Definition: Vertical Line

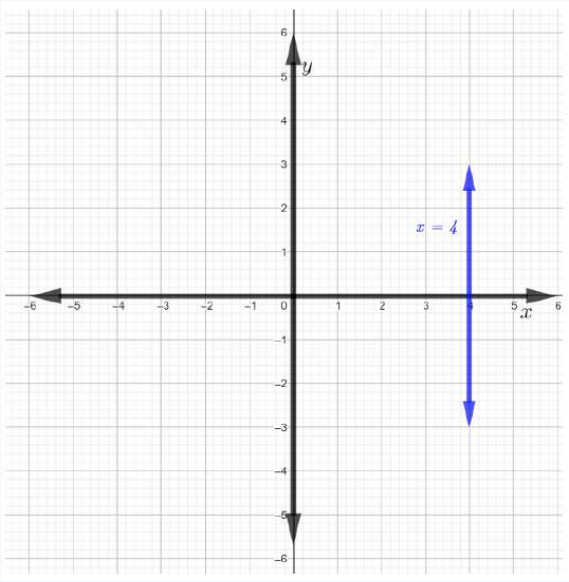
The equation of a **vertical line** is of the form $x = c$, where c is any real number. The vertical line will always intersect the $x-$axis at the point $(c, 0)$. The slope of a vertical line is *undefined*.

Example 7.4.1

Find the slope of the line $x = 4$ and graph the line.

Solution

$x = 4$ is the graph of a vertical line as shown in the figure below.



To find the slope of line $x = 4$ choose any two distinct points on the line. Let the points be $(4, -1)$ and $(4, 3)$. Using the slope of a line formula,

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Now, if 4 is divided by 0, this is equivalent to asking the question, "what number times zero gives 4?" the answer is, there is no such number. Division by zero is undefined, and the slope of the vertical line $x = 4$ is **undefined**.

Definition: Horizontal Line

The equation of a **horizontal line** is of the form $y = k$, where k is any real number. The horizontal line will always intersect the $y-$axis at the point $(0, k)$. The slope of a horizontal line is *Zero*.

Example 7.4.2

Find the slope of the line that passes through the points $(-3, -2)$ and $(4, -2)$. Plot the points and graph the line that passes through them.

Solution

Use the slope of the line formula. Thus,

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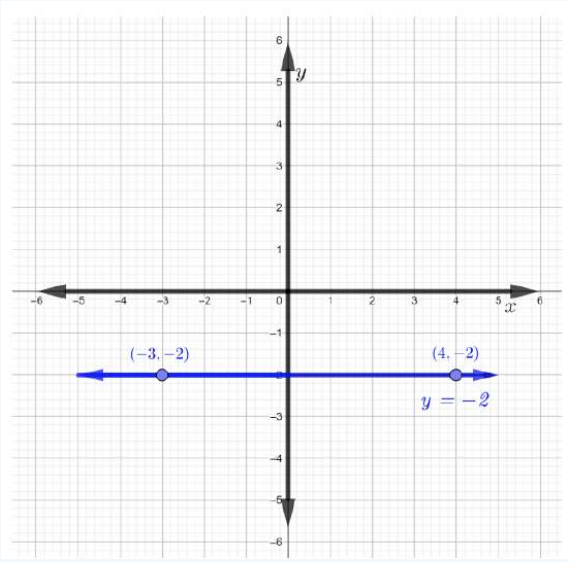
Therefore, the line that passes through the two given points is a horizontal line, with slope equal zero, as shown in the figure below.

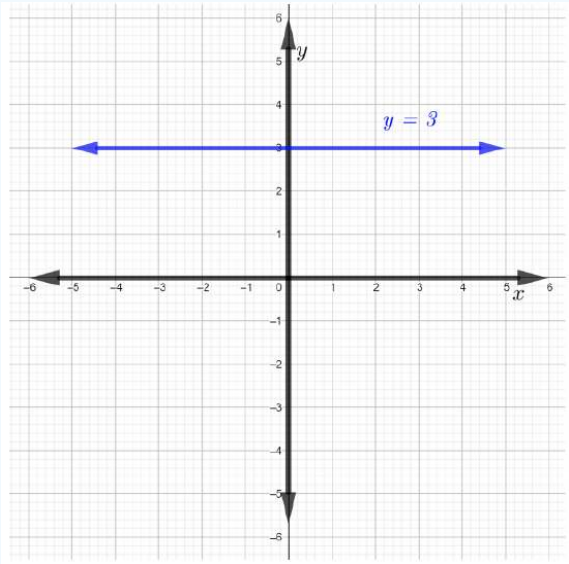


Example 7.4.3

Graph the line $y - 3 = 0$ and find its slope.

Solution

The line $y - 3 = 0$ can be written as $y = 3$ (add 3 to both sides of the equation). The line $y = 3$ is a horizontal line, as shown in the figure below.



Now, to find the slope, choose any two distinct points on the line $y = 3$. Consider points $(0, 3)$ and $(3, 3)$. Thus,

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Therefore, the slope of the given line is $m = 0$.

Exercise 7.4.1

Find the slope of each line.

1. $x = -\frac{1}{2}$
2. $y - 1 = 0$
3. $x + 7 = 10$
4. $y + 2 = -9$
5. Find the slope of the line that passes through the points $(-4, 1)$ and $(2, 1)$. Plot the points and graph the line that passes through them.
6. Find the slope of the line that passes through the points $(-3, 5)$ and $(-3, -7)$. Plot the points and graph the line that passes through them.

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7.5: Forms of the Equation of a Line

The previous section explained the equations of vertical and horizontal lines. Now discover three more forms of the equations of a line, namely, the **Slope-Intercept Form**, the **Point-Slope Form**, and the **Standard Form**.

7.5.1: Slope-Intercept Form of the Equation of a Line

Definition: Slope-Intercept Form

The Slope-Intercept Form of the equation of a line is of the form:

$$y = mx + b$$

Where m is the slope of the line and $(0, b)$ is the $y-$intercept.

Note that the y-intercept is the point where the line intersect the $y-$axis, that is when $x = 0$.

Example 7.5.1

Write an equation of the line with the given slopes and y-intercepts.

a. slope = 5; $y-$intercept $(0, \frac{1}{2})$

b. slope = $-\frac{5}{6}$; $y-$ intercept $(0, -\frac{3}{4})$

Solution

a. $m = 5$ and $b = \frac{1}{2}$

The equation of a line is of the for $y = mx + b$. Thus,

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Therefore, $y = 5x + \frac{1}{2}$ is the equation of the line with the given slope and y-intercept.

b. Given $m = -\frac{5}{6}$ and $b = -\frac{3}{4}$

Thus,

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Therefore, $y = -\frac{5}{6}x - \frac{3}{4}$ is the equation of the line with the given slope and y-intercept.

Example 7.5.2

Identify the slope and $y-$intercept then, use them to graph each line.

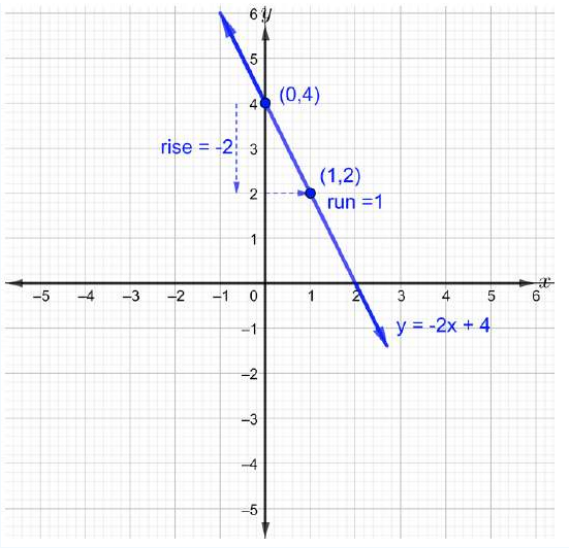
a. $y = -2x + 4$

b. $5y - 3x = 10$

Solution

a. Notice that the given linear equation is in the slope-intercept form. So, $m = -2$ or equivalently, $m = -\frac{2}{1}$ and $b = 4$

m is the slope of the line, then $m = \frac{\text{rise}}{\text{run}} = -\frac{2}{1}$. To graph the line, plot at least two points. Start at the $y-$intercept $(0, 4)$ and move down 2 unit then move to the right 1 unit to plot the second point. Now join the two points with a straight line as shown in the figure below.



b. Notice that it is not clear how to identify the slope and y-intercept in this given linear equation because it is not in the slope-intercept form. Thus, solve for y to have the equation in the slope-intercept form as follows,

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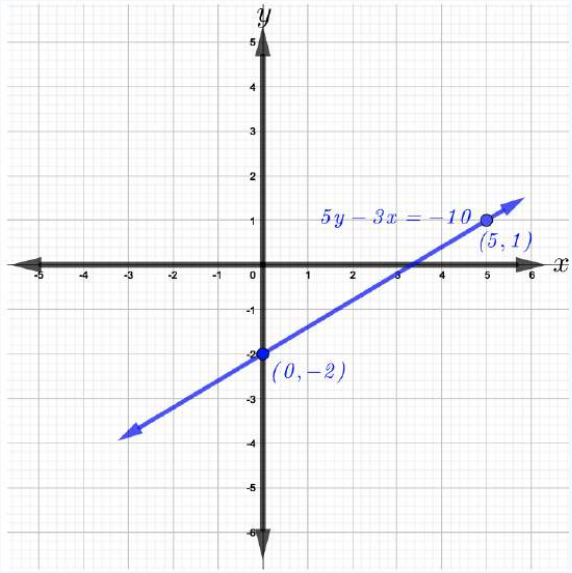
Now, $m = \frac{3}{5}$ and $b = -2$. Start by plotting the y-intercept $(0, -2)$ then move 3 units upward and 5 units to the right and plot the second point which is $(5, 1)$. Now, join the two points, namely, $(0, -2)$ and $(5, 1)$ to get the graph of the line shown in the figure below.



Exercise 7.5.1

Write an equation of a line with the given slope and y-intercept.

a. slope: 2 y-intercept: $(0, \frac{3}{4})$
b. slope: $\frac{5}{7}$ y-intercept: $(0, -6)$
c. slope: $-\frac{1}{2}$ y-intercept: $(0, -\frac{7}{11})$

Exercise 7.5.2

Identify the slope and y-intercept then use them to graph each line.

a. $y = 5x - 3$
b. $2y = -6x + 1$

7.5.2: Point-Slope Form of the Equation of a Line

Definition: Point-Slope Form

The **Point-Slope Form** of the equation of a straight line is:

$$y - y_1 = m(x - x_1)$$

Where m is the slope of the line and (x_1, y_1) is any point on the straight line.

Example 7.5.3

Find the equation of each line passing through the given point and given slope.

a. Slope 3 and point $(-1, 8)$
b. Slope $-\frac{5}{2}$ and point $(\frac{4}{3}, \frac{1}{3})$

Solution

a. To find the equation of the line through the point $(-1, 8)$ with slope $m = 3$, use the point-slope form as follows:

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Therefore, $y = 3x + 11$ is the equation of the line with the given slope and point. The line is in the slope-intercept form.

b. Similar to part a, use the Point-Slope Form as follows:

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Therefore, $y = -\frac{5}{2}x + 3$ is the equation of the line through the give point and the given slope.

Example 7.5.4

Find an equation of the line given points $(2, 4)$ and $(-3, 9)$.

Notice that earlier in this chapter it explained how to find an equation of a line given a slope and y-intercept. This chapter also explained how to find an equation of a line given any point on the line and a slope. So, in both methods, the slope is given.

Solution

To find an equation of a line given any two points on the line, first find the slope using the slope of the line formula. After, apply the point-slope form with any of the given points. First, use the two points to find the slope of the line. Let $(x_1, y_1) = (2, 4)$ and $(x_2, y_2) = (-3, 9)$. Then,

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Now the slope has been found so next find the equation of the line using any one of the given points. Thus, $m = -1$ and consider using point $(2, 4)$.

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Therefore, $y = -x + 6$ is the equation of the line passing through the giving point and has the slope-intercept form.

Exercise 7.5.3

Find the equation of each line passing through the given point and has the given slope.

a. The slope $-\frac{5}{2}$ and the point $(3, 0)$.

b. The slope $\frac{1}{2}$ and the point $(-2,-3)$.

Exercise 7.5.4

Find an equation of the line given the following Points.

a. $(-9,-3)$ and $(6,-2)$
b. $(4,1)$ and $(-2,2)$

7.5.3: Standard Form of the Equation of a Line (AKA General Form of a Linear Equation)

Definition: Standard Form

The **standard form** of a non-vertical line is in the form

$$Ax + By = C$$

Where A is a positive integer, B and C are integers with $B \neq 0$.

Example 7.5.5

Graph each line of the following equations:

a. $4x - 3y = 6$
b. $\frac{1}{2} - y + 1 = 0$

Note that the x-intercept is the point where the line intersects the x-axis. That is, when $y = 0$. Thus, the x-intercept is a point of the form $(a, 0)$, where a is any real number.

Solution

a. The equation $4x - 3y = 6$ is in standard form. To graph the line of the given equation it may be possible to use more than one method. For example, solving for y to get the equation in slope-intercept form, then, graph the line. It's also possible to find two points, then graph the line. The two easiest points to find quickly are the x and y intercepts. So, this method is recommended.

To find the x-intercept, set $y = 0$ in the given equation and solve for x as follows,

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Hence, the x-intercept is the point $(\frac{3}{2}, 0)$

Now, to find the y-intercept, set $x = 0$ as follows,

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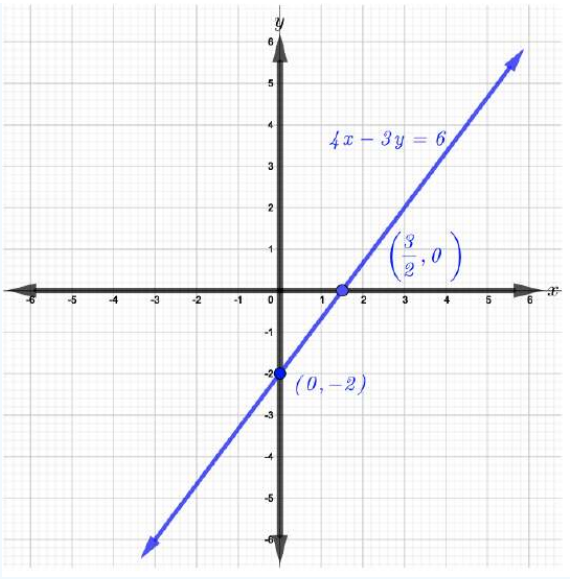
Now, plot the points $(\frac{3}{2}, 0)$ and $(0,-2)$ and graph the straight line that passes through them as shown in the figure below.



The equation $\frac{1}{2}x - y + 1 = 0$ is not in standard form. So, subtract 1 from both sides of the equation to have $\frac{1}{2}x - y = -1$ which is now in standard form.

Again, similar to part b, find the x and y-intercepts. First, find the x-intercept by setting $y = 0$ and solve for x as follows.

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Thus, the x-intercept is the point $(-2, 0)$.

Now, set $x = 0$ to find the y-intercept, as follows,

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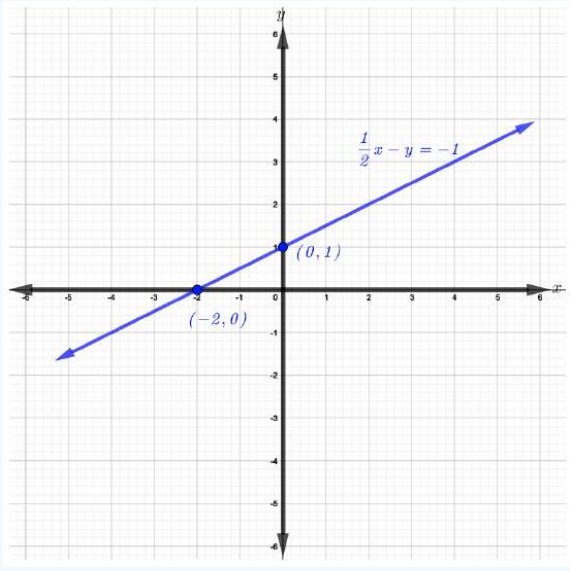
Hence, the y-intercept is $(0, 1)$.

Plot the x and y-intercepts, $(-2, 0)$ and $(0, 1)$, then graph the straight line that passes through them as shown in the figure below.



There is no homework for this section.

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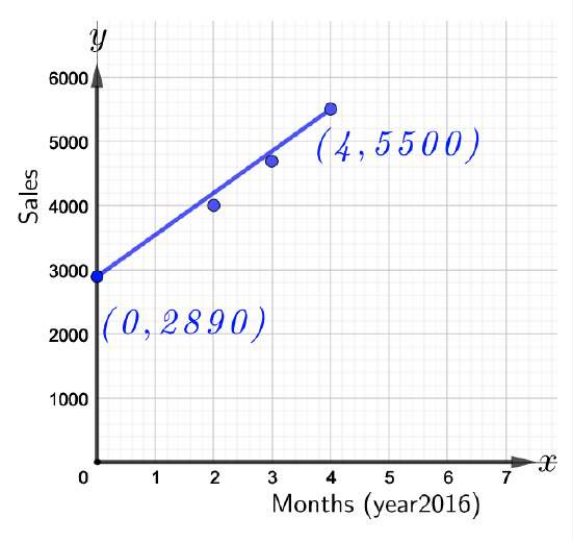
7.6: Applied Examples

To better understand the concepts learned in this chapter, apply them to real-life situation and every day problems.

Example 7.6.1

The total sales of phone covers in a popular phone store (in thousands) are shown in the figure below. The month of March, 2016, corresponds to $x = 0$.



a. Refer to the figure above to write an equation of the line that models the data. What does the slope indicate?
b. Use the equation in part a to approximate the sales of phone covers in the month of November, 2016.

Solution

a. Referring to the figure above, notice that there are two points, $(0, 2890)$ and $(4, 5500)$, that lie on the given line. Use the two points to first find the slope. Thus,

$$m = \frac{5500 - 2890}{4 - 0} = \frac{2610}{4} = 652.5$$

Thus, the slope is 652.5. Since the slope is positive, it indicates an increase. Hence, the slope indicates that the sales of phone covers have increased by about $\$652.5$ per month from March 2016 to July 2016.

Use the slope and the y-intercept $(0, 2890)$ to write the equation of the line in slope-intercept form as follows,

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b. Now, it is given that $x = 0$ corresponds to the month of March and the problem asks to approximate the sales of phone covers in the month of November 2016. Thus, $x = 8$ corresponds to the month of November 2016. Substitute $x = 8$ in the equation found in part a. Thus,

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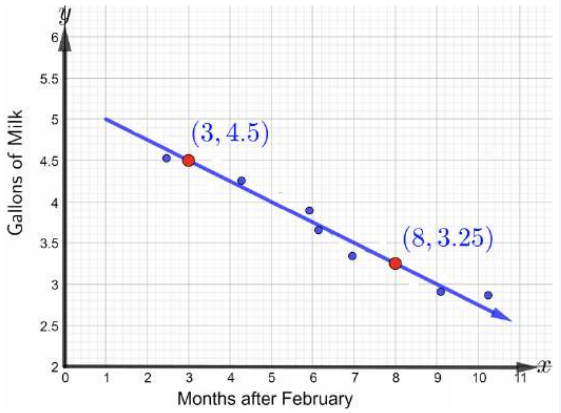
Therefore, the sales of phone covers in the month of November 2016 were approximately $\$8,110$.

Example 7.6.2

A mother was concerned if her toddler is consuming enough calcium. The toddler's main calcium intake was in the form of milk. The mother recorded the data for nine months to monitor the toddler's intake of milk. The data is represented in a scattered diagram that shows the amount of milk that the toddler consumed each month for nine months, from February to October as shown in the figure below.



a. Refer to the figure above to write an equation of the line to model the given data using the two labeled points. Write the equation in a slope-intercept form. What can be said about the slope?
b. Use the equation of the line found in part a to predict the amount of mild that the toddler will consume in December.

Solution

a. To write the equation of the line, first find the slope using the two labeled points, $(3, 4.5)$ and $(8, 3.25)$. Thus,

$$m = \frac{3.25 - 4.5}{8 - 3} = -\frac{1.25}{5} = -0.25$$

The slope is negative means that the toddler's milk consumption is decreasing by about 0.25 gallons of milk each month.

Since the y-intercept is not given, it will not be possible to use the slope intercept form. Instead, use the slope and either of the two labeled points in the point-slope form to find the equation of the line. Thus,

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Thus, $y = -0.25x + 5.25$ is the equation of the line that represents the data given in the figure above and is in slope-intercept form.

b. To predict how many gallons of milk will the toddler consume in December, first find x that corresponds to the month of December. It is given that $x = 0$ corresponds to the month of February. Starting at the month of March with $x = 1$, $x = 10$ corresponds to the month of December. Substitute $x = 10$ in the equation of the line found in part a and solve for y as follows,

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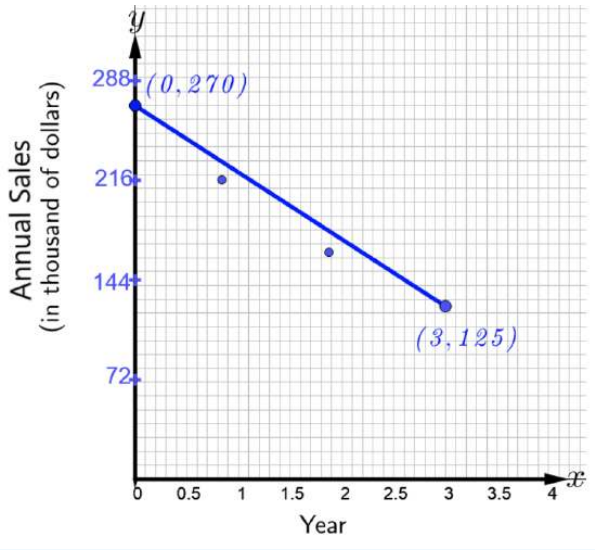
Therefore, the toddler will consume about 2.75 gallons of milk in December.

Exercise 7.6.1

The annual sales of a certain model of printers in a famous electronics store are shown in the figure below, where the year 2012 corresponds to $x = 0$.

https://math.libretexts.org/@go/page/45200



a. Refer to the figure above to write an equation of the line that models the data. What does the slope indicate?
b. Use the equation of the line found in part a to approximate the sales of the printers in the year 2016.

Exercise 7.6.2

John bought an ice cream cone that is 11.2 cm tall. It was really hot, the ice cream started to melt at a rate of 2 cm per minute. John got interested at how fast his ice cream was melting and wanted to know how much ice cream would be left after 2.5 minutes.

a. Find an equation of the line that models the data in slope-intercept form.
b. Determine how much ice cream is left after 2.5 minutes.

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CHAPTER OVERVIEW

8: Polynomial Operations

8.1: Polynomial Addition and Subtraction (and Combining Like Terms)

8.2: Polynomial Multiplication

8.3: Factoring and Finding Polynomial Solutions (Zeroes)

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8.1: Polynomial Addition and Subtraction (and Combining Like Terms)

Definition: Like Terms

To add and subtract polynomials, combine **like terms**. Like terms have the same variables with the same exponents. The coefficients of the terms may be different.

Be careful when subtracting, to distribute the subtraction (think of it as an addition of (-1) times the polynomial).

Example 8.1.1

Add or Subtract the Polynomials:

a. $(-6a^3+5a^2-7a-9)+(3a^3+5a^2+a+8)$
b. $(4x^2-3)+(3x^2-8x+7)$
c. $(3x^2-4x+6)-(2x^2-x-9)$
d. $(-4x^3+5x^2+15)-(2x^2-4x+9)$

Solution

a. Illegal pream-token (&)

b. Illegal pream-token (&)

c. Illegal pream-token (&)

d. Illegal pream-token (&)

Exercise 8.1.1

Add or Subtract the Polynomials

1. $(5x^2+8)-(x^2+4x+3)$
2. $(x^3-14x^2)-(-4x^3+5x^2+8)$
3. $(6x^2+7x-9)+(-9x^2+2)$
4. $(x^3+6x^2-8x+14)+(9x^3-7x^2+5x-11)$
5. $(3x-4)-(2x^2-3x-9)$
6. $(2x^2+x+3)-(5x^2-1)$

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8.2: Polynomial Multiplication

Polynomials can be classified as:

- Monomials if they contain one term.
- Binomials if they contain two term.
- Trinomials if they contain three term.
- Polynomials if they contain three or more terms.

There are no examples or homework in this section.

8.2.1: Multiplication of Two Monomials

Definition: Multiplying Two Monomials

To **multiply two monomials**, multiply the terms together by adding the exponents and multiplying the numeric coefficients.

Example 8.2.1

Multiply two monomials:

a. $(3x^2)(6x^3)$
b. $(4x)(x)$
c. $(-2x^3)(-7x^4)$

Solution

a. Illegal pream-token (&)

b. Illegal pream-token (&)

c. Illegal pream-token (&)

Exercise 8.2.1

Multiply two monomials:

1. $(-3x^4)(9x^7)$
2. $(2x)(2x)$
3. $(-4x^7)(5x^5)$
4. $(-6x^2)(-x^2)$

8.2.2: Multiplication of a Polynomial by a Monomial

Definition: Multiplying a Polynomial By a Monomial

To **multiply a polynomial by a monomial**, multiply all terms of the polynomial by the monomial. Keep any subtractions in the original polynomial with the term following the subtraction as the sign of the coefficient of the term.

Example 8.2.2

Multiply a polynomial by a monomial:

a. $3x^2(15x^2 - 5x)$
b. $-7x(3x^2 - 2x + 9)$
c. $5x(4x^3 - 2x^2 + x - 3)$

Solution

a. Illegal pream-token (&)

b. Illegal pream-token (&)

c. Illegal pream-token (&)

Exercise 8.2.2

Multiply a polynomial by a monomial:

1. $(-6x)(x^2-3)$
2. $(3x^4)(2x^2-x-5)$
3. $(-4x^5)(x^4-3x^3+3x^2-x-7)$
4. $(x^2)(-x^3-12)$

8.2.3: Multiplication of Two Binomials

Definition: Multiplying Two Binomials

To **multiply two binomials**, use the FOIL technique to multiply: first terms, outer terms, inner terms and the last terms. FOIL ensures that all terms in the first binomial are multiplied with all terms in the second binomial. The order of multiplication of terms does not matter since multiplication is commutative. Take care to combine any like terms to fully simplify the solution.

Example 8.2.3

Multiply two binomials:

a. $(3x-4)(2x+5)$
b. $(5x^2-2)(5x^2+2)$
c. $(7x^3-4x^2)(x-5)$

Solution

a. Illegal pream-token (&)

b. Illegal pream-token (&)

c. Illegal pream-token (&)

Exercise 8.2.3

Multiply two binomials:

1. $(2x-3)(6x+5)$
2. $(3x^2-4)(3x^2+4)$
3. $(-4x^5-2)(7x^3+3)$
4. $(2x-7)(3x-8)$

8.2.4: Multiplication of Two Polynomials

Definition: Multiplying Two Polynomials

To **multiply two polynomials**, use the distributive property to multiply every term in the first polynomial by every term in the second polynomial. Like terms are then combined to simplify the solution.

Example 8.2.4

Multiply two polynomials:

a. $(2x+5)(3x^2-6x+9)$
b. $(2x^2+4x-5)(3x-2)$

c. $(x^2 - x + 3)(2x^2 + 6x - 1)$

Solution

a. Illegal pream-token (&)

b. Illegal pream-token (&)

c. Illegal pream-token (&)

Exercise 8.2.4

Multiply two polynomials:

1. $(x^2 - 2x - 1)(2x^2 - 7x - 8)$
2. $(3x^2 - 5)(x^2 + 4x - 3)$
3. $(4x^3 - 2x + 1)(6x^2 + 3)$
4. $(2x^3 - 3x + 4)(2x^2 - 8x + 2)$

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8.3: Factoring and Finding Polynomial Solutions (Zeroes)

There are several ways to find solutions of polynomials which are trinomials of the form $ax^2 + bx + c = 0$. These solutions are also called the real zeroes of polynomials.

1. **Trial and Check Factoring Method:** With this method, the goal is to create two binomials that when multiplied together, result in the given trinomial. This method can be very difficult when the given trinomial has large values of a and c. Once the factoring is complete find all real zeroes by using the zero factor property and setting each factor equal to 0 and solve for x.
2. **Factor by Grouping Factoring Method:** With this method, the goal is to create four terms by splitting the middle term into two terms, whose coefficients have a product of $a * c$ and have a sum of b. The order of the center terms does not matter. Once the four terms are created, pair the first two terms with parentheses, pair the second two terms with parentheses, and factor out the GCF from both pairs. The resultant repeated binomial is one factor, and the GCF factors combine to make the second binomial. This is the easiest method to use on any factorable trinomial of the form $ax^2 + bx + c$, but may have a bit of a learning curve. Once the factoring is complete find all real zeroes by using the zero factor property and setting each factor equal to 0 and solve for x.
3. **The Quadratic Formula:** The Quadratic Formula can be used to find the real zeroes of a factorable trinomial. Please see the Table of Contents to find the section explaining how to use the Quadratic Formula.

Example 8.3.1

Factor the expressions using any of the methods discussed in this section (these example problems will demonstrate the Factor by Grouping method):

a. $4x^2 - 3x - 10$
b. $8x^2 - 2x - 3$
c. $12x - 14x^3 + 22x^2$
d. $\dfrac{(x^2+1)^2(-2) + (2x)2(x^2+1)(2x)}{(x^2+1)^4}$
e. $\dfrac{(2x+1)^{\frac{1}{2}} - (x+2)(2x+1)^{-\frac{1}{2}}}{2x+1}$

Solution

a. Illegal pream-token (&)

b. Illegal pream-token (&)

c. Illegal pream-token (&)

d. Illegal pream-token (&)

e. Illegal pream-token (&)

Exercise 8.3.1

Factor using any method discussed in this section:

1. $5x^2 - 23x - 10$
2. $8x^2 + 2x - 3$
3. $3x^2 - 7x - 6$
4. $10x^2 + 13x - 5$
5. $12x^5 - 17x^4 + 6x^3$
6. $\dfrac{(2x^2-1)^2(-2) + (2x)2(2x^2-1)(2x)}{(2x^2-1)^4}$
7. $\dfrac{2(2x-3)^{\frac{1}{3}} - (x-1)(2x-3)^{-\frac{2}{3}}}{2x-3^{\frac{2}{3}}}$

8.3.1: Quadratic Formula

Definition: Quadratic Formula

The **Quadratic Formula** is used to solve (or find the zeroes) of a Polynomial (Quadratic Equation) of degree 2 that is in the form $ax^2 + bx + c = 0$. The Quadratic Formula is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where a, b, and c are the coefficients of the standard form of a Quadratic Equation, $ax^2 + bx + c = 0$.

Example 8.3.2

For the following functions, find all zeros of f using the Quadratic Formula. Express the final answer as exact answers (in radical form) and also as decimals, rounded to the thousandths place.

a. $f(x) = -2x^2 + 4x - 1$
b. $f(x) = x^3 - 3x^2 - 4x$

Solution

a. Set $f(x) = 0 : -2x^2 + 4x - 1 = 0$. This function is written in the form $ax^2 + bx + c = 0$, with $a = -2$, $b = 4$ and $c = -1$.

Replacing a, b and c in the Quadratic Formula with these values:

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b. The function $f(x) = x^3 - 3x^2 - 4x$ is a cubic function. Factor out x from all three terms before using the Quadratic Equation on the trinomial factor: $x(x^2 - 3x - 4) = 0$, with $a = 1$, $b = -3$ and $c = -4$.

Don't forget that the x that was factored out is a root, namely $x = 0$.

Replacing a, b and c in the Quadratic Formula with these values:

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There are three solutions, or roots of the cubic function $f(x) = x^3 - 3x^2 - 4x : x = 0$, $x = -1$ and $x = 4$.

Exercise 8.3.2

For the following functions, find all zeros of f using the Quadratic Formula. Express the final answer as exact answers (in radical form) and also as decimals, rounded to the thousandths place.

1. $f(t) = 9t^3 - 18t^2 + 6t$
2. $f(x) = x^5 - 4x^4 - 32x^3$
3. $f(x) = 18 - 3x - 2x^2$
4. $f(x) = 12x^2 + 11x - 5$
5. $f(x) = 3x^2 - 6x + 2$

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CHAPTER OVERVIEW

9: Rational Expressions

9.1: Simplify Rational Expressions
9.2: Multiply Rational Expressions
9.3: Add and Subtract Rational Expressions
9.4: Rationalize Algebraic Fractions

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9.1: Simplify Rational Expressions

Definition: Rational Expression

A **rational expression** is written as a quotient of polynomials

$$\frac{P(x)}{Q(x)}$$

where $P(x)$ and $Q(x)$ are polynomials in one variable x.

To simplify a rational expression, factor both the numerator and the denominator, and remove common factors from both the numerator and the denominator. A simplified rational expression has only one division, and a single numerator and denominator. If the expressions cannot be factored, then the rational expression cannot be simplified.

Example 9.1.1

Simplify the rational expressions:

a. $\dfrac{x^2+2x-3}{x^2+4x+3}$

b. $\dfrac{(x^2+1)^2(-2)+(2x)(2)(x^2+1)(2x)}{(x^2+1)^4}$

c. $\dfrac{(x^2+1)\frac{1}{2}(x^{-\frac{1}{2}})-(2x)(x^{\frac{1}{2}})}{(x^2+1)^2}$

Solution

a. Illegal pream-token (&)

b. Illegal pream-token (&)

c. Illegal pream-token (&)

Exercise 9.1.1

Simplify the rational expressions:

1. $\dfrac{2x^2+3x-2}{2x^2+5x-3}$
2. $\dfrac{(t^2+4)(2t-4)-(t^2-4t+4)(2t)}{(t^2+4)^2}$
3. $\dfrac{(2)(x-4)(x^2+4x+4)}{(x+2)(x^2-16)}$
4. $\dfrac{12x^2+19x-21}{12x^2+38x-40}$

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9.2: Multiply Rational Expressions

Definition: Multiplying Rational Expressions

To **multiply rational expressions**, multiply the numerator expressions and multiply the denominator expressions. Then, if possible, simplify by factoring both the numerator and denominator and removing common factors.

Try to use factor by grouping when working with a 4-term polynomial.

Note: Common denominators are not needed when multiplying rational expressions!

Example 9.2.1

Multiply the rational expressions:

a. $\frac{10t}{6t-12} * \frac{20t-40}{30t^3}$

b. $\frac{7x+14}{2x^2-8} * \frac{x^2+3x-10}{14x+21}$

c. $\frac{3x^3-24}{2x^2-14x+20} * \frac{4x^3-20x^2+3x-15}{3x^2+6x+12}$

Solution

a. Illegal pream-token (&)

b. Illegal pream-token (&)

c. Illegal pream-token (&)

Exercise 9.2.1

Multiply the rational expressions:

1. $\frac{x^2+4x+3}{2x^2-x-10} * \frac{2x^2+4x^3}{x^2+3x} * \frac{x}{x^2+3x+2}$
2. $\frac{x^2+2x-15}{x^2-4x-45} * \frac{x^2-5x-36}{x^2+x-12}$
3. $\frac{x^2+3x-40}{x^2+2x-35} * \frac{x^2+3x-18}{4x^2-5x-32x+40}$

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9.3: Add and Subtract Rational Expressions

Definition: Add or Subtract Rational Expressions

To **add or subtract rational expressions**, think of this as fractions with variables. A common denominator (called the LCD) is needed for addition and subtraction.

9.3.1: Find the LCD/LCM

Definition: LCD/LCM

To find the **LCD**, first fully factor all denominators. Build the LCD from the factors found in all denominators. Multiply each factor the greatest number of times it occurs in either expression. If the same factor occurs more than once in both expressions, multiply the factor the greatest number of times it occurs *in either expression.* It will be called the **LCM** in this section (Least Common Multiple), because there are no fractions in these problems.

Example 9.3.1

a. (x^2-2x-3) and $(x^2+2x-15)$
b. (x^2-9) and $(2x^2-5x-3)$
c. (x^2+x-2) and (x^2+4x+4)

Solution

a. Illegal pream-token (&)

b. Illegal pream-token (&)

c. Illegal pream-token (&)

Exercise 9.3.1

Find the LCM:

1. $(3x^2-13x+4)$ and (x^2-16)
2. $(2x^2+x-3)$ and (x^2-2x+1)
3. $(x-1)$ and (x^2-4x-5)
4. $(6x^2-23x+20)$ and $(4x^2-25)$

9.3.2: Subtract Rational Expressions and Simplify to a Single Rational Expression

Definition: Add or Subtract Rational Expressions Using the LCD

Rational expressions are fractions with variables (also known as algebraic fractions). To add or subtract rational expressions, first find the common denominator (the LCD), then add or subtract the numerators, keeping the same (common) denominator. Finally, factor and simplify by removing common factors from the numerator and denominator if possible.

Example 9.3.2

Add or Subtract and Simplify:

a. $\dfrac{2x}{2x-1}-\dfrac{2x}{2x+5}$
b. $\dfrac{4}{x^2-9}-\dfrac{5}{x^2-6x+9}$
c. $\dfrac{x}{1+x}+\dfrac{2x+3}{x^2-1}$

Solution

a. Illegal pream-token (&)

b. Illegal pream-token (&)

c. Illegal pream-token (&)

Exercise 9.3.2

Add or Subtract and Simplify:

1. $\frac{x}{x^2+1}+\frac{24x^3}{x3+2}$
2. $\frac{x}{1-x}+\frac{2x+3}{x^2-1}$
3. $\frac{5}{x+3}+\frac{x^2-4x-21}{x^2-9}$
4. $\frac{39x+36}{x^2-3x-10}-\frac{23x-16}{x^2-7x+10}$

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9.4: Rationalize Algebraic Fractions

Definition: Denominator of Rational Expressions

If the **denominator** of a rational expression contains sums or differences involving radicals, it is good form to always rationalize the denominator by multiplying the numerator and denominator by the conjugate of the denominator.

The **conjugate** of the denominator contains the same terms, but opposite operations (addition or subtraction).

Example 9.4.1

Rationalize the Denominator and Simplify:

a. $\dfrac{1}{1-\sqrt{x}}$

b. $\dfrac{1}{\sqrt{x}-\sqrt{y}}$

c. $\dfrac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}}$

Solution

a. Illegal pream-token (&)

b. Illegal pream-token (&)

c. Illegal pream-token (&)

Exercise 9.4.1

Rationalize the Denominator and Simplify:

1. $\dfrac{x}{1-\sqrt{x}}$
2. $\dfrac{1}{1-\sqrt{x}}$
3. $\dfrac{2\sqrt{x}}{\sqrt{x}-1}$
4. $\dfrac{x-1}{\sqrt{x}-1}$

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CHAPTER OVERVIEW

10: Inequalities

10.1: Properties of Inequalities

10.2: Solving and Graphing Inequalities, and Writing Answers in Interval Notation

10.3: Rational Inequalities

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10.1: Properties of Inequalities

Here are some important properties of inequalities:

Properties of Inequalities

If a, b, and c are real numbers, then:

Transitive Property if $a < b$ and $b < c$ then $a < c$

Addition Property if $a < b$ then $a + c < b + c$

Subtraction Property if $a < b$ then $a - c < b - c$

Multiplication Property (Multiplying by a positive number) if $a < b$ and $c > 0$ then $ac < bc$

Multiplication Property (Multiplying by a negative number) if $a < b$ and $c < 0$ then $ac > bc$

Division Property (Dividing by a positive number) if $a < b$ and $c > 0$ then $\frac{a}{c} < \frac{b}{c}$

Division Property (Dividing by a negative number) if $a < b$ and $c < 0$ then $\frac{a}{c} > \frac{b}{c}$

There are no examples or homework in this section.

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10.2: Solving and Graphing Inequalities, and Writing Answers in Interval Notation

To solve and graph inequalities:

1. Solve the inequality using the Properties of Inequalities from the previous section.
2. Graph the solution set on a number line.
3. Write the solution set in interval notation.

Example 10.2.1

Solve the inequality, graph the solution set on a number line and show the solution set in interval notation:

a. $-1 \leq 2x - 5 < 7$
b. $x^2 + 7x + 10 < 0$
c. $-6 < x - 2 < 4$

Solution

a. Illegal pream-token (&)

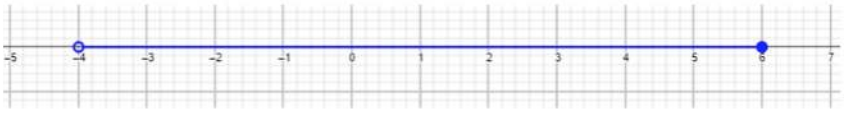
b. Illegal pream-token (&)

Solution set for $x > -5$.

Solution set for $x - 2$.

Illegal pream-token (&)

c. Illegal pream-token (&)



Exercise 10.2.1

Solve the inequalities, graph the solution sets on a number line and show the solution sets in interval notation:

1. $0 \leq x + 1 \leq 4$
2. $0 < 2(x - 1) \leq 4$
3. $6 < 2(x - 1) < 12$
4. $x^2 - 6x - 16 < 0$
5. $2x^2 - x - 15 > 0$

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10.3: Rational Inequalities

Solving rational inequalities involves finding the zeroes of the numerator and denominator, then using these values to investigate solution set regions on the number line.

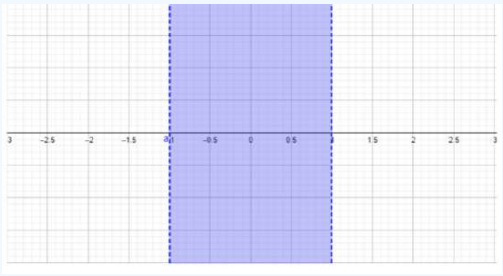
Example 10.3.1

Solve the inequalities and write the solution sets in interval notation:

a. $\frac{x-1}{x+1} \geq 0$
b. $\frac{2x-3}{x+1} \leq 0$
c. $\frac{x+2}{x-2} \geq 0$

Solution

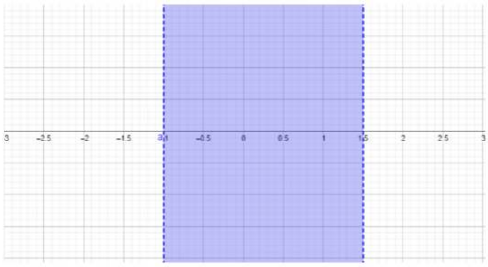
a. Illegal pream-token (&)



The zeroes divide the number line into 3 regions, $x < -1$, $-1 < x < 1$, $x > 1$

Illegal pream-token (&)

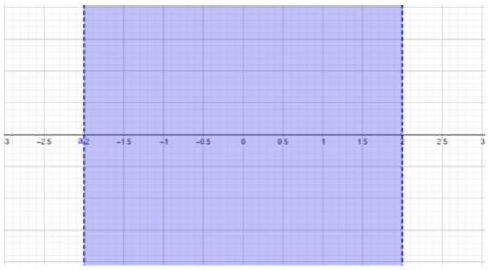
b. Illegal pream-token (&)



The zeroes divide the number line into 3 regions, $x < -1$, $-1 < x < 1.5$, $x > 1.5$

Illegal pream-token (&)

c. Illegal pream-token (&)



The zeroes divide the number line into 3 regions, $x < -2$, $-2 < x < 2$, $x > 2$

Illegal pream-token (&)

Exercise 10.3.1

1. $\frac{x+3}{x-2} \geq 0$
2. $\frac{x-2}{x-1} \leq 0$
3. $\frac{2x-1}{x+2} \leq 0$
4. $\frac{2x-3}{x+1} \geq 0$

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